

Factoring; Add-Trial & Error

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So far we have factored using the Distributive Property and the Difference of Two Squares. Today, we'll look at TRINOMIALS and use the Addition Method or Trail & Error.

The general form of a quadratic equation is

$$ax^2 + bx + c = 0$$

The "a" represents the coefficient of the quadratic term. When $a = 1$, we'll use the **Addition Method** of factoring.

When we multiplied binomials using FOIL, the short-cut was to get the middle term and multiply to get the constant term as shown.

$$(x + 3)(x + 5) = x^2 + 8x + 15$$

Now, to factor we go backwards. To factor $x^2 + 7x + 12$, I would find all the numbers that multiplied together result in 12.

They are

12, 1
6, 2
4, 3

Which of those would add to 7, the coefficient of the linear term? Therefore the factors of $x^2 + 7x + 12$ are $(x + 3)(x + 4)$

EXAMPLE Factor $x^2 - 7x - 30$

Since the coefficient of the quadratic term is one, we use the Addition Method.

We find the numbers multiplied together that result in -30 **AND** add up to -7.

30, -1	-30, 1
15, -2	-15, 2
10, -3	-10, 3
6, -5	-6, 5

All the products are -30, the only factors that add up to a -7 is -10 and 3. Therefore the factors are $(x - 10)(x + 3)$

If the coefficient of the quadratic term is not one, we use **Trail & Error** to factor the trinomial.

The name Trail and Error suggests we try different factors to see if they work, if they don't we try others.

Pick factors that work for the quadratic and constant terms, then check to see if when multiplied out using FOIL, we get the linear term

EXAMPLE Factor $12x^2 + 56x + 9$

Picking factors for 12 and 9, I have the following choices.

$(12x - 9)(x + 1)$	$(12x - 1)(x - 9)$
$(6x - 1)(2x - 9)$	$(4x - 1)(3x - 9)$
$(4x - 3)(3x - 3)$, etc	

Because of space considerations, I did not list all the possibilities. Using FOIL, which of those will add up to 56? Hopefully you identified $(6x + 1)(2x + 9)$ since the sum of the Outer and Inner numbers is 56.

EXAMPLE Factor $6x^2 + 19x + 10$.

Since the coefficient of the quadratic term is **not** one, I should use Trial & Error. But there is an alternative.

Stay with me. If I multiply the constant term (10) by the coefficient of the quadratic term (6), I get 60.

What numbers multiply together to give you 60 and add up to 19? That's right 15 and 4. So

$$x^2 + \underline{19x} + 60 = 6x^2 + \underline{15x} + \underline{4x} + 10$$

By multiplying the coefficient of the leading term and the constant, that gives me a hint how I might break the linear term into a sum.

Now factoring that by grouping the first two terms together and the last two terms by the Distributive property, I have

$$6x^2 + 15x + 4x + 10$$

Taking a $3x$ out of the first two terms and a 2 out of the last two terms, we have

$$3x(2x + 5) + 2(2x + 5)$$

Taking the $(2x + 5)$ out of both terms leaves me with $(2x + 5)(3x + 2)$.

Sure enough, $(2x + 5)(3x + 2) = 6x^2 + 19x + 10$