Solving Systems of Equations

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When you solve two equations at the same time, we call that solving the equations "simultaneously," or solving a system of equations. What we are doing is looking for a solution (answer, ordered pair) that satisfies both equations. The geometric interpretation is we are finding the point where two or more lines intersect. A point that is on both lines.

There are two methods of solving equations simultaneously, LINEAR COMBINATION and SUBSTITUTION. Either method will produce correct answers. Generally, I like to use linear combination.

The strategy for solving equations by LINEAR COMBINATION is to add the equations together in such a way as to eliminate a variable. Then you solve the resulting equation and voila – Your done. Piece of cake.

Let's write an algorithm that will help us accomplish that strategy.

1. If necessary, multiply one or both of the equations by a number that will make the coefficients of one of the variables the same, but opposite in sign. i. e. $2x$, $-2x$ or $5y$, $-5y$.
2. Add the two equations together (that eliminates a variable).
3. Solve the resulting equation.
4. Plug that value back into either of the original equations to find the value of the other variable.

EXAMPLE:

$$3x + 10y = 2$$
$$x - 2y = 6$$

First, I have to determine what variable I want to get rid of. If I multiply the bottom equation by (-3) and add the equations, that will get rid of the x's. OR I could multiply the bottom equation by 5 and add the equations and that will get rid of the y's. It DOES NOT MATTER WHICH YOU CHOOSE.

I'll multiply the bottom equation by (-3).

$3x + 10y = 2$		$3x + 10y = 2$
$x - 2y = 6$	mult by -3	$\underline{-3x + 6y = -18}$
	Adding	$16y = -16$
	Solve	$y = -1$

Now we plug that value back into either of the original equations. The second equation looks simpler, so I'll plug it in there.

$x - 2y = 6$ Plug in $y = -1$

$x - 2(-1) = 6$ Simplify

$x + 2 = 6$

$x = 4$

The answer is the ordered pair (4, -1). That's the point where these two lines intersect. Again, the strategy is to add the equations together to eliminate a variable. Yes, I can hear you. You love math, math is your life!

EXAMPLE:

$$2x + 3y = -4$$
$$5x - 2y = 9$$

First, I have to determine which variable is easier to get rid of. Oh boy, in the last equation, all I had to do was multiply one equation by a number to get the coefficients the same. This time I'll have to multiply both equations. I can multiply the top by -5 and the bottom by 2, that will get rid of the x's OR I could multiply the top by 2 and the bottom by 3, that will get rid of the y's. I'll choose to get rid of the y's.

I'll multiply the top equation by 2 and the bottom equation by 3.

$2x + 3y = -4$	mult by 2	$4x + 6y = -8$
$5x - 2y = 9$	mult by 3	$\underline{15x - 6y = 27}$
	Adding	$19x = 19$
	Solve	$x = 1$

Now plug that value for x back into either of the original equations. Using the top equation:

$$2(1) + 3y = -4$$
$$3y = -6$$
$$y = -2$$

The answer is the ordered pair (1, –2). If you chose to get rid of the x's first, you would have gotten the same answer.

The other method used to solve systems of equations is SUBSTITUTION. The strategy is to solve one of the equations in terms of the other, substitute that into the OTHER equation and solve the resulting equation, then play Mr. Plug-in as in the last problem.

The algorithm for SUBSTITUTION

1. Solve one of the equations for one of the variables.
2. Substitute that expression in the OTHER equation.
3. Solve the resulting equation.
4. Plug that value into either of the original equations to find the other variable.

Let's redo the first example using this method.

EXAMPLE: $3x + 10y = 2$
$x - 2y = 6$

First, determine which equation has the easiest variable to solve for, x and the second equation is cake.

$3x + 10y = 2$
$x - 2y = 6$, solving for x, $x = 6 + 2y$

Everywhere you see an x in the OTHER equation, substitute 6 + 2y. The other equation is:

$3\mathbf{x} + 10y = 2$	Substitute $6 + 2y$
$3\mathbf{(6 + 2y)} + 10y = 2$	Simplify
$18 + 6y + 10y = 2$	
$18 + 16y = 2$	
$16y = -16$	
$y = -1$	Solve

Plug that back into either of the original equations, you find x = 4. JUST LIKE BEFORE.

Generally I'll use LINEAR COMBINATIONS to solve systems of equations. I only consider SUBSTITUTION when one of the coefficients of one of the variables is 1, as in the first example. When the coefficients are not 1, as in example 2, I'll use linear combination.

Try this one both ways: $2x + y = 11$
$3x - 2y = -1$

Solution (3, 5)