Chapter 6 Polynomials

Polynomials; Add/Subtract

Polynomials – sounds tough enough. But, if you look at it close enough you'll notice that students have worked with polynomial expressions such as $6x^2 + 5x + 2$ since first grade. The only difference is they have letters (x's) instead of powers of ten. They have been taught that 652 means 6(100) + 5(10) + 2(1). They have been taught the six tells them how many hundreds they have, the five how many tens, and the two how many ones are 9in the number.

$$6(100) + 5(10) + 2(1) \rightarrow 6(10)^2 + 5(10) + 2(1) \rightarrow 6x^2 + 5x + 2$$

In the polynomial expression $6x^2 + 5x + 2$, called a ***tri***nomial because there are three terms, the six tells how many x^2 s there are, the 5 tells you how many x's, and the two tells you how many ones.

The point is polynomial expressions in algebra are linked to what's referred to as *expanded notation* in grade school. It's not a new concept.

In grade school we teach the students how to add or subtract numbers using place value. Typically, we have them line up the numbers vertically so the ones digits are in a column, the tens digits are in the next column and so on, then we have them add or subtract from right to left.

In algebra, we have the students line up the polynomials the same way, we line up the numbers, the x's, and the x^2 's, then perform the operation as shown below.

$$\begin{array}{l} 3x^2 + 4x + 3 \rightarrow 343 \\ \underline{2x^2 + 3x + 5 \rightarrow 235} \\ 5x^2 + 7x + 8 \rightarrow 578 \end{array}$$

Notice when adding, we added "like" terms. That is, with the numbers, we added the hundreds column to the hundreds, the tens to the tens. In algebra, we added the x^2 's to the x^2 's, the x's to the x's

In algebra, the students can add the expressions from right to left as they have been taught or left to right. If the students understand place value, this could lead students to add columns of numbers more quickly without regrouping by adding numbers from left to right.

The students would have to add the hundreds column, then add to that the sum of the tens column, and finally the sum of the ones column

Example Add, in your head 341 + 214 + 132

You have 300 + 200 + 100, that's 600, adding the tens, we have 40 + 10 + 30 which is 680, and finally adding 1 + 4 + 2 or 7, the sum is 687.

We can add, subtract, multiply, and divide polynomials using the same procedures we learned in elementary school.

In the first grade you learned to add the ones column to the ones column, the tens to the tens, hundreds to hundreds. We use that same concept to add polynomials, we add numbers to numbers, x's to x's, and x^2's to x^2's. We call that combining like terms.

EXAMPLE $(3x^2 + 2x - 4) + (5x^2 - 7x - 6)$

Combining like terms, we have

$$3x^2 + 5x^2,\ \ 2x - 7x,\ \ -4 - 6$$

$$8x^2 - 5x - 10$$

Subtraction of polynomials is just as easy. We will look at subtraction as adding the opposite, In other words $5 - 2$ is the same as $5 + (-2)$. Using the reasoning, when we subtract polynomials, we will add the opposite,

EXAMPLE $(3x^2 + 2x - 4) - (5x^2 - 7x - 6)$

Changing the signs and combining like terms, we have

$$3x^2 - 5x^2,\ \ 2x + 7x,\ \ -4 + 6$$

$$-2x^2 + 9x + 2$$

Oh yes, math is a blast! Remember to change **ALL** the signs in the subtraction, then add.

All too often students do not realize a rule or procedure they are learning is nothing more than a shortcut. For that reason, math is like magic for many students. If we take the time to develop the patterns, students would not be so easily befuddled.

Simplify

1. $(3x^2 + 5x + 9) + (2x^2 + 4x + 10)$
2. $(3x^2 + 5x + 7) + (8x^2 + 4x + 3)$
3. $(5x^2 - 6x + 5) + (4x^2 - 5x - 8)$
4. $(x^2 + 7x - 9) + (10x^2 - 7x - 8)$
5. $(3x^2 + 5x + 9) - (2x^2 + 4x + 10)$
6. $(3x^2 + 5x + 7) - (8x^2 + 4x + 3)$
7. $(5x^2 - 6x + 5) - (4x^2 - 5x - 8)$
8. $(x^2 + 7x - 9) - (10x^2 - 7x - 8)$

Sec. 2 Polynomials: Multiplication

Polynomials are multiplied the very same way students learned to multiply in third and fourth grades. Unfortunately, they don't realize it. They hear this shortcut called FOIL, and they become FOILed.

In grade school, students are taught to line up the numbers vertically. In algebra, students typically multiply horizontally. Let's look at multiplying two 2-digit numbers and compare that to multiplying two binomials

$$\begin{array}{r} 32 \\ 21 \\ \hline 32 \\ 64 \\ \hline 672 \end{array} \qquad\qquad\qquad \begin{array}{r} x+3 \\ x+1 \\ \hline x+3 \\ x^2+3x \\ \hline x^2+4x+3 \end{array}$$

Notice the same procedure is used.

If students were to look at a number of examples, they may be able to see a pattern develop that would allow them to multiply binomials in their head.

Look at the numbers in the problems, look at the numbers in the answers. Do you see a pattern?

$$(x+3)(x+1) = x^2 + 4x + 3$$

$$(x+5)(x+2) = x^2 + 7x + 10$$

$$(x+4)(x+5) = x^2 + 9x + 20$$

If you examine these problems long enough, you might notice the middle term of the trinomial comes from adding the numbers and the constant term comes from multiplying those same numbers

What do you think $(x+3)(x+2)$ would be equal? Since 3 +2 2 = 5 and 3 x 2 = 6, if you said $x^2 + 5x + 6$, then you saw the pattern.

Notice – in all those examples, the coefficient of the linear term, the number in front of the x was ONE. When that occurs, we add the numbers to get the middle term and multiply to get the constant.

Simplify

1. $(x + 5)(x + 3)$
2. $(x + 7)(x+3)$
3. $(x + 6)(x + 2)$
4. $(x + 5)(x + 10)$
5. $(x + 8)(x - 5)$
6. $(x - 4)(x + 6)$
7. $(x - 5)(x - 3)$
8. $(x - 10)(x - 5)$

Let's look at an example where the coefficients of the linear terms are not ONE.

Example (**3**x + 4) (**2**x + 5)

Many students would multiply these by applying the shortcut we just found, they would add to get the middle term and multiply to get the last number.

$$(3x + 4)(2x + 5) = 6x^2 + 9x + 20$$

However, doing the problem the long way using the fourth grade procedure, let's see what happens.

$$\begin{array}{r} 3x + 4 \\ \underline{2x + 5} \\ 15x + 20 \\ \underline{6x^2 + 8x \phantom{{}+20}} \\ 6x^2 + 23x + 20 \end{array}$$

Using the shortcut, the linear term we thought would be 9, doing it the long way, the middle term is 23.

That's a problem. Where did we go wrong in the shortcut of adding and multiplying?

Well, it appears the shortcut of adding and multiplying only works when the coefficients of the linear terms are ONE. The last example, the coefficients are not ONE.

Is there a way of finding this product using a shortcut? Again, the only term that did not work was the middle term. We thought it would have been a 9, it turned out to be 23. Where'd the 23 come from?

Looking at the problem, the 23x came from adding the 15x and 8x. Where'd the 15x and 8x come from? Again, looking at the problem, the 15x came from multiplying 3x by 5, the 8x came from multiplying the 2x by 4.

Going back to the original problem – $(3x + 4)(2x + 5)$

The 3x and the 2x represent the **First** numbers in each parentheses.

The 3x and 5 represent the numbers on the **Outside** of the parentheses,

The 4 and 2x are the **Inside** numbers, and the

4 and 5 represent the **Las**t numbers in each parentheses.

$$\mathbf{(3x + 4)\ (2x + 5)}$$

Multiplying the First #'s,	3x and 2x	$= 6x^2$
the Outers,	3x and 5	$= 15x$
the Inners,	4 and 2x	$= 8x$
the Last,	4 and 5	$= 20$

Combining like terms, we have $6x^2 + 23x + 20$

With practice, you will be able to multiply polynomials by the FOIL method in your head.

Let's try one using a schematic for FOIL.

$(5x + 3)\ (4x + 7)$ **Outside numbers – 35x**

Inside numbers – 12x

$20x^2 + 35x + 12x + 21$

$20x^2 + 47x + 21$

Again, most students would have gotten the first and last terms without much difficulty, it's the **O I**, the middle term that causes students heartburn.

We can use the FOIL method to multiply two digit numbers.

Example 32 x 21

$$\begin{array}{r} 32 \\ \underline{\times 21} \end{array}$$

Multiplying the first numbers, we have 3 x 2 = 6

Multiplying the Last numbers, we have 1 x 2 = 2

So the answer should look like 6 _ 2

The middle number should come from multiplying the Outside numbers and adding that to the Inside numbers. That is 1 x 3 + 2 x 2 = 7.

32 x 21 = 672

With a little practice, you will be able to multiply two digit numbers in your head. I picked an easy example – I didn't have to carry.

Sec. 3 Special Products -Factoring

Some patterns allows students to compute in their head. Most students know how to multiply by powers of ten in their head. For instance,

72 x 10 = 720 34 x 100 = 3,400 65 x 1000 = 65,000

Students are often told when they multiply by ten, just add a zero. Well, 72 + 0 still equals 72. What was meant was to add a zero to the end of the number. While the rule for multiplying by powers of ten don't make sense, the pattern is easy to see and implement. Because we can multiply by powers of ten in our head, we think of that as a special product.

Another shortcut, mental math, is multiplying by eleven. Look at the next few problems and see if you can see a pattern.

25 x 11 = 275 35 x 11 = 385 71 x 11 = 781

Notice the first and last digits are in the original problem. How was the middle digit found?

Upon careful inspection, you might notice that the middle digit could be found by adding the two digits together.

So, in order to multiply 42 x 11, the first number in the answer is 4, the last number is 2. To find the middle number, we add 4 and 2. Therefore, 42 x 11 = 4 6 2. Try 45 x 11. Hopefully you got 495.

If the digits you multiply result in carrying, then you have a slight variation. If I multiplied 75 x 11, the first number in the product should be 7, the last should be 5. The answer should look like 7 _ 5. If you added 7 and 5, the result would be 12. If I put 12 as the middle number, my product would be 7125. That would not make sense since the answer should be around 700.

Since I can not put 12 in, what I will need to do is put the middle number as 2 and carry the 1.

75 x 12 = 8 2 5

After looking at the following multiplications, some people would be able to square numbers in their head. Let's look:

25 x 25 = 625 35 x 35 = 1225 65 x 65 = 4225 95 x 95 = 9025

Do you see a pattern that would allow you to multiply 75 x 75?

In algebra, you will study special products, its nothing more than mental math described by patterns.

Let's look at some:

$$(a-b)(a+b)=a^2-b^2$$

What that pattern allows you to do is multiply numbers like 32 x 28 in your head.
32 x 28 becomes $(30+2)(30-2)=30^2-2^2$ or 900 – 4 = 896.

Compute 47 x 53 in your head. If you recognize the pattern, both numbers are three from 50, I can rewrite that as $(50-3)(50+3)=50^2-3^2$ or 2500 – 9 = 2491.

Not only does that pattern allow us, with practice to multiply in our head, it will also help us factor polynomials – if we take the time to recognize the pattern.

Let's look at another special product:

$$(a+b)^2=a^2+2ab+b^2$$

That pattern allows us to square numbers mentally. $35^2=(30+5)(30+5)$. Using that pattern, we have $30^2+2x(30x5)+5^2$ which is 900 + 300 + 25 or 1225.

Another example $43^2 \rightarrow (40+3)(40+3)$. Using the pattern, I would have

$$40^2+2x(40x3)+3^2=1600+240+9 \text{ or } 1849$$

Using subtraction, we have $(a-b)^2=a^2-2ab+b^2$, which is almost the same pattern we just used. Using this pattern, we can simplify 37^2 mentally.

$$37^2=(40-3)^2$$
$$(40-3)^2=40^2-2x(40x3)+3^2$$
$$=1600-240+9$$
$$=1369$$

Recognizing the patterns in these computations will help students recognize the same patterns in algebra that will help them factor algebraic expressions and solve higher degree equations.

Factoring polynomials is not much different than finding the prime factors of composite numbers done in earlier grades. What has to happen in both situations is students have to look at the problem at hand, then using their background knowledge, identify patterns, then select a method to rewrite the composite number as a product.

Let's look at a couple of examples. If I asked a student to find the prime factors of 117, the first thing many students would do is check to see if 117 is divisible by numbers they are very familiar, such as 2, 5, and 10. Since 117 is not even and does not end in 0 or 5, they students will deduce that they can't use those numbers.

Now, they will have to look for other ways of finding factors. Hopefully, rather than using trial and error and dividing, they would use the rules of divisibility. Since the sum of the digits of 117 is 9, we know that 117 is divisible by 3 and by 9.

So 117 = 9 x 13, factoring the 9, we have 117 = $3^2 \times 13$

Polynomial factoring uses the same type of reasoning. That is, you look at a polynomial and determine if there are common factors, much like a student would have done by checking if a number was divisible by 2, 5, or 10. In algebra, we would factor out those common factors using the Distributive Property.

Once that was accomplished, a student would look for a method to factor the rest of the polynomial. With composite numbers, students determined which rule of divisibility might apply.

Once students' factored using the Distributive Property, they would then determine if the polynomial was a binomial, trinomial, or other.

If the expression was a binomial, students would try to use the Difference of Two Squares to factor.

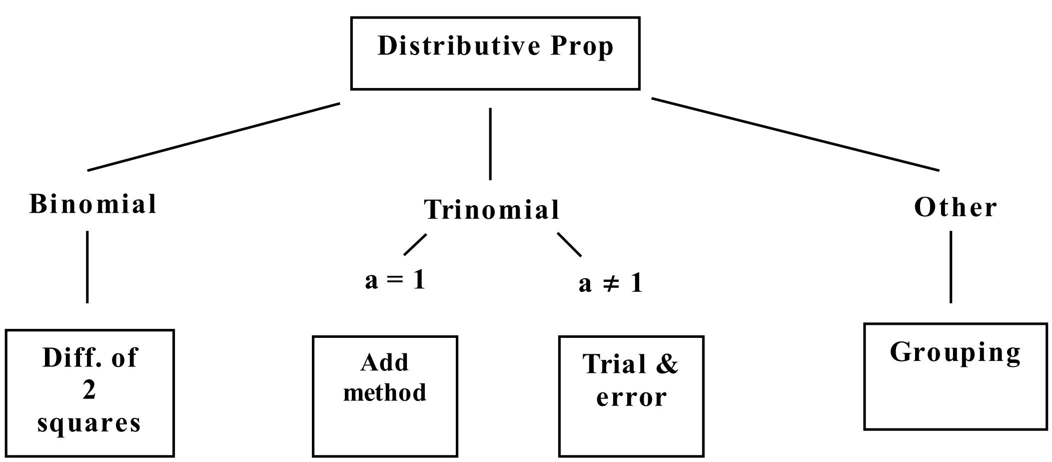
If the expression was a trinomial, students would look at the leading coefficient, often referred to as a, and if $a = 1$, they would factor using the Addition Method.

If they had a trinomial and $a \neq 1$, the students would try a method called Trial & Error.

And finally, if the expression was not a binomial or trinomial, students would try and factor by Grouping.

Finding prime factors of composite numbers is made a lot easier by knowing the Rules of Divisibility. Factoring polynomials is made a lot easier by knowing the five factoring methods; Distributive Property, Difference of Two Squares, Addition Method, Trial & Error, and Grouping.

Factoring is the process of changing a polynomial expression that is essentially a sum into an expression that is essentially a product.



Factoring is used to simplify algebraic expressions and solve higher degree equations.

The diagram suggests that you always factor by first trying to use the **Distributive Property.** After that, you then determine if you have a binomial, trinomial or other.

If you have a BINOMIAL, we'll try anduse the method called the **Difference of 2 Square**.

If you have a TRINOMIAL and the coefficient if the quadratic term (number in front of the squared term) is one, we'll use the **Addition Method**. If the coefficient of the quadratic term is NOT one, then we will use **Trial & Error** to factor.

If I don't have a binomial or trinomial, then I will use **Grouping**.

Let's look at factoring using the **Distributive Property**.

$$\mathbf{a(b + c) = ab + ac}$$

To factor, we look for numbers of letters that appear in each term of a polynomial.

EXAMPLE Factor $4x + 12$

Is there a number or letter that appears on both terms? Hopefully you realized there was a 4 in both terms. Taking the 4 out, we have

$$4x + 12 = 4(x + 3)$$

EXAMPLE Factor $4x + 12\,xy$

I have a 4x in both terms, factoring that we have

$$\mathbf{4x + 12xy = 4x(1 + 3y)}$$

Now let's see what happens if I have a binomial. The diagram suggests I use the **Difference of Two Squares.**

$$\mathbf{a^2 - b^2 = (a - b)(a + b)}$$

I use the Difference of Two Squares if I have a binomial, both terns being perfect squares, and a " – " sign separates them.

To factor a binomial using the Difference of Two Squares, you take the square root of each term, write them twice, as shown below, put a "+" sign between one and a " – " between the other.

EXAMPLE Factor $x^2 - 9$

$$x^2 - 9 = (x + 3)(x - 3)$$

EXAMPLE Factor $49x^2 - 81y^2$

The square roots of $49x^2$ and $81y^2$ are 7x and 9y. Therefore we have

$49x^2 - 81y^2$ - $(7x + 9y)(7x - 9y)$

Factor completely using the Distributive property or Difference of Two Squares

A	B	C
$3x + 6$	$5y^2 - 15y$	$2x^2 - 4x$
$5xy - 10xy^2$	$6xy^2z^3 - 12xy2$	$8x^2 - 4x^3y^2$
$x^2 - 25$	$y^2 - 16$	$z^2 - 100$
$3x^2 - 12$	$3x^2 - 75$	$2x^3 - 32x$
$3x^2y - 27xy$	$5y^2 - 5$	$x - 1$
$x^{16} - 1$	$x^2 - 81$	$x^2 + 25$
$5x^3y - 20xy$	$y^2 + 16$	$2x^2 - 100$

So far we have factored using the Distributive Property and the Difference of Two Squares. Now, we'll look at TRINOMALS and use the Addition Method or Trail & Error.

Sec. 4 Addition Method

The general form of a quadratic equation is

$$\mathbf{ax^2 + bx + c = 0}$$

The "a" represents the coefficient of the quadratic term. When a = 1, we'll use the **Addition Method** of factoring.

When we multiplied binomials using FOIL, the short-cut was to get the middle term by adding and multiply to get the constant term as shown below.

$3+5 = 8$ and $3x5 = 15$ $(x + 3)(x + 5) = x^2 + 8x + 15$

Now, to factor polynomials whose leading coefficient is 1, we go backwards.

To factor $x^2 + 7x + 12$, I would find all the numbers that multiplied together result in 12.

They are

12, 1
6, 2
$\boxed{4, 3}$

Which of those would add to 7, the coefficient of the linear term? 4 and 3. Therefore the factors of $x^2 + 7x + 12$ are $(x + 3)(x + 4)$

EXAMPLE Factor $x^2 - 7x - 30$

Since the coefficient of the quadratic term is one, we use the Addition Method.

We find the numbers multiplied together that result in –30 **AND** add up to –7.

30, -1	-30, 1
15, -2	-15, 2
10, -3	$\boxed{-10, 3}$
6, -5	-6, 5

All those products are –30, the only factors that add up to a –7 are –10 and +3. Therefore the factors are $(x – 10)(x + 3)$

Factor using the Addition Method

$x^2 + 9x + 20$	$x^2 + 8x + 12$	$x^2 + 13x + 42$
$x^2 + 10x + 16$	$x^2 + 5x + 6$	$x^2 + 11x + 10$
$x^2 – x – 20$	$x^2 + x – 20$	$x^2 – 2x – 24$
$x^2 – 4x – 21$	$x^2 – x – 2$	$x^2 + 7x – 30$

$x^2 - 7x + 10$	$x^2 - 9x + 20$	$x^2 - 3x + 2$
$x^2 - 7x + 6$	$x^2 - 12x + 32$	$x^2 - 10x + 24$
$x^2 + 11x + 28$	$x^2 - 3x - 28$	$x^2 - 15x + 50$
$2x^2 + 18x + 40$	$3x^2 - 6x - 72$	$5x^2 - 5x - 10$

Sec. 5 Trial & Error

If the coefficient of the quadratic term is not one, we use **Trail & Error** to factor the trinomial.

The name Trail and Error suggests we try different factors to see if they work, if they don't we try others.

Pick factors that work for the quadratic and constant terms, then check to see if when multiplied out using FOIL, we get the linear term

EXAMPLE Factor $12x^2 + 56x + 9$

Picking factors for 12 and 9, I have the following choices.

(12x 9)(x 1) (12x 1)(x 9)
(6x 1)(2x 9) (4x 1)(3x 9)
(4x 3)(3x 3), etc

Because of space considerations, I did not list all the possibilities. Using FOIL, which of those will add up to 56 – the coefficient of the linear term? Hopefully you identified $(6x + 1)(2x + 9)$ since the sum of the Outer and Inner numbers is 56.

So $12x^2 + 56x + 9 = (6x + 1)(2x + 9)$

EXAMPLE Factor $6x^2 + 19x + 10$.

Since the coefficient of the quadratic term is not one, I should use Trial & Error.

I find factors for the quadratic term $6x^2$, 3x and 2x, then I find factors for the constant 10, 5 and 2. Because I picked those factors, I know I will have $6x^2 + ?x + 10$. So I have the first and last term. But will those factors give me a 19 for the linear term?

Checking the outers and inners using FOIL, I do get 19.

Therefore $6x^2 + 19x + 10 = (3x + 2)(2x + 5)$

Sec. 6 Trial and Error Alternative

But there is an alternative to that guess and check method we call Trial & Error.

Stay with me. On the last example, if I multiply the constant term (10) by the coefficient of the quadratic term (6), I get 60.

What numbers multiply together give me 60 AND add up to the linear term 19?

Hopefully, you identified 15 and 4. Now, I will rewrite my original expression.

$6x^2 + 19x + 10$ as $6x^2 + 15x + 4x + 10$

Now, I will factor the first two terms using the Distributive property and then the second two terms using the Distributive Property.

$$6x^2 + 19x + 10 = 3x\mathbf{(2x + 5)} + 2\mathbf{(2x + 5)}$$

There is a (2x + 5) in both terms, taking that out using the distributive property, we have

$$3x\mathbf{(2x + 5)} + 2\mathbf{(2x + 5)} = (2x + 5)(3x + 2)$$

Example Factor completely $10x^2 + 21x + 9$

Multiply the constant (9) by the leading coefficient (10), you get 90.

What numbers multiply together give you 90 and add up to the coefficient of the linear term (21)?

If you answered 15 and 6, we are in business.

We'll rewrite our original expression $10x^2 + 21x + 9$ as $10x^2 + 6x + 15x + 9$

Factoring using the Distributive Property, we have

$$10x^2 + 6x + 15x + 9 = 2x\mathbf{(5x + 3)} + 3\mathbf{(5x + 3)}$$

Now I have a (5x +3) in each term, factoring out the (5x + 3), we have

$$2x\mathbf{(5x + 3)} + 3\mathbf{(5x + 3)} = \mathbf{(5x + 3)(2x +3)}$$

Factor using Trial & Error or the Alternative Method.

$6x^2 + 9x + 3$

$8x^2 + 14x + 5$

$6x^2 + 19x + 10$

$12x^2 + 20x + 3$

$12x^2 + 28x – 5$

$6x^2 – 5x – 21$

$5x^2 + 58x - 24$ $5x^2 - 2x - 24$

$4x^2 + 23x + 15$ $4x^2 - 7x - 15$

Sec. 7 Factoring, combined

We have learned to factor polynomials using the Distributive Property, Difference of Two Squares, Addition Method, and Trial & Error. Now, do you know which method to use by looking at the polynomial?

The greatest problem students encounter while factoring is determining which method to use. Teachers need to take the time to teach the students to compare and contrast. To some students, all polynomials look alike. They have to be explicitly taught to differentiate between the problems so they can use the correct methodology.

If we go back to our diagram, we see we should try to factor first by using the Distributive Property. That will make the numbers more manageable.

Look at the following polynomials and select the method that should be used to factor it.

a. $8y^2 + 2y - 3$
b. $x^2 - 25$
c. $z^2 -7x - 12$
d. $2x^2 + 18x$
e. $a^3 - 3a^2 + 9a - 27$

Let's see how you did. Answer a. is Trial & Error because the coefficient of the quadratic term is NOT one. Answer b. is the Difference of Two Squares because you have a binomial, both terms are perfect squares, and it is a difference. Answer c, is the Addition Method because the coefficient of the quadratic term is one. Answer d. is the Distributive Property and answer e. is Grouping since it is not a binomial or trinomial.

If you can discriminate between the polynomials, that's half the battle. Because once you know what method to use, you just follow the procedures you have already learned.

Now let's see how you go about factoring polynomials by **GROUPING**.

To be able to factor using Grouping requires you to know how to factor using other methods. Then using those methods, we group terms together.

EXAMPLE Factor $a^3 - 3a^2 + 9a - 27$

I can not use the Distributive Property, nor do I have a binomial of trinomial, that's a pretty good indication that I have to factor by Grouping.

The question is, how do I group? The first two and the last two terms, the first three terms and the fourth term? The first and third term and the second and fourth?

Well, we are going to group them to try and find common factors.

Notice, if I took an a^2 out of the first two terms and a 9 out of the third and fourth terms, I have a common factor of $(a – 3)$

$$a^3 - 3a^2 + 9a – 27 = a^2 \mathbf{(a – 3)} + 9 \mathbf{(a – 3)}$$

Factoring out the $(a – 3)$ from both terms, we have

$$\mathbf{(a – 3)}(a^2 + 9)$$

Yep, that was fun.

EXAMPLE Factor $x^2 - 6x + 9 - y^2$

This again is a Grouping problem. In this case, I recognize $x^2 - 6x + 9$ as a perfect square. Since I see this pattern, I will group the first three terms together and factor.

$$x^2 - 6x + 9 - y^2 = (x -3)^2 - y^2$$

If I looked at this result, I might recognize that as the Difference of Two Squares.

Using that method, I take the square root of each term and write them as factors.

$$[(x – 3) \quad y] \{(x – 3) \quad y]$$

I then put a "+" sign in one of the factors, a " – " sign in the other. That gives me

$$[(x – 3) = y] [(x – 3 – y]$$

It's important that you can recognize special products when factoring by grouping. That information will give you a clue on how to group the terms.

Factor the following expressions using the appropriate method.

1. $2x^2 + 7x + 5$
2. $3x^2 + 10x + 7$
3. $5x^2 + 7x + 2$
4. $8x^2 – 10x + 3$
5. $6x^2 – 17x + 5$
6. $10x^2 – 19x + 7$
7. $x^2 + x – 6$
8. $x^2 + x – 12$
9. $x^2 + x – 30$
10. $2a^2 – 7a – 4$
11. $3x^2 – 10x – 25$
12. $2x^2 – x – 3$

13. $x^2 + 7x + 10$

14. $b^2 + 5b + 6$

15. $a^2 + 4a + 3$

16. $y^2 - 10y + 16$

17. $m^2 - 9m + 20$

18. $d^2 - 11d + 30$

19. $5y^2 - 5$

20. $4x^2 - 100$