Word Problems – 2 variables

Word problems, you know you love them. A while ago we solved word problems in one variable. The trick you might remember was to read the word problem to determine what type it was, then you read it again to find out what you are looking for, then you read it again to build relationships and you read it again to make an equation. Finally, you solved the equation.

The point I want to make is you have to read a word problem 5 or 6 times just to get the information you need to solve it. If you read the problem only once or twice, then you are not giving yourself the opportunity to be successful. Enough said.

Now, when you encounter a word problem, you are going to be faced with a choice. The choice will be to solve using one variable or to solve using more than one variable.

To solve using more than one variable, you read the question to find out what you are looking for and label those.

Let’s look at an example.

EXAMPLE:

The sum of two consecutive even numbers is 62. Find the numbers.

First, we’ll do the problem in one variable like we have done before. We are looking two numbers.

I will call them #1 and #2. The smallest we’ll call x. The second was is a consecutive even number, how do I get that? I add 2 to the first number.

#1 - x
#2 - x + 2

Now read the question again to find an equation. The sum of the numbers is 62. That means 
#1 + #2 = 62. Plugging those numbers in, I have,

x + (x + 2) = 62

Now, solve for x.

2x + 2 = 62
2x = 60
x = 30  The first number is 30, the second is 32.

Doing that same problem using more than one variable, I again identify what am I looking for. I will use #1 and #2 again. Now, I’ll call the first number x and the second number y.

#1 - x
#2 - y

Now this is very important. Since I have 2 variables I must now find two equations. If I had three variables, I would need to three equations.

What do I know about these numbers? Their sum is 62. Mathematically we write:

x + y = 62

That’s one equation, are there any other relationships? The numbers are consecutive even integers. How can I express that mathematically? The second number minus the first must be 2. Therefore we have
\[ y - x = 2 \]

Putting those two equations together, we have this system.

\[
\begin{align*}
  x + y &= 62 \\
  y - x &= 2
\end{align*}
\]

or

\[
\begin{align*}
  x + y &= 62 \\
  -x + y &= 2
\end{align*}
\]

Now I solve that system of equations by linear combination of substitution. Easy as pie.

The nice thing about solving equations in more than one variable is I call my unknown \( x, y, z, \) etc., then I look for relationships and solve the resulting systems of equations.

I must have at least the same number of equations as variables. Otherwise I can’t solve the system.