

DECIMALS

DECIMALS are special fractions whose denominators are powers of 10.

Since decimals are special fractions, then all the rules we have already learned for fractions should work for decimals. The only difference is the denominators for decimals are powers of 10; i.e., $10^1, 10^2, 10^3, 10^4$, etc. ... Students normally think of powers of 10 in standard form; 10, 100, 1000, 10,000.

In a decimal, the numerator is the number to the right of the decimal point. The denominator is not written, but is implied by the number of digits to the right of the decimal point. The number of digits to the right of the decimal point is the same as the number of zeros in 10, 100, 1000,

Therefore, one place is tenths, two places is hundredths, three places is thousandths, and so on.

- Examples:**
- 1) $.56 - 2 \text{ places} - \frac{56}{100}$
 - 2) $.532 - 3 \text{ places} - \frac{532}{1000}$
 - 3) $3.2 - 1 \text{ place} - 3\frac{2}{10}$

The correct way to say a decimal numeral is to:

- 1) Forget the decimal point.
- 2) Say the number.
- 3) Then say its denominator and add the suffix "ths".

- Examples:**
- 1) $.53 - \text{Fifty-three hundredths}$
 - 2) $.702 - \text{Seven hundred two thousandths.}$
 - 3) $.2 - \text{Two tenths}$
 - 4) $5.63 - \text{Five and sixty-three hundredths.}$

When there are numbers on both sides of the decimal point, the decimal point is read as "and". You say the number on the left side, the decimal point is read as "and", then say the number on the right said with its denominator.

Example Write 15.203 in word form

Fifteen and two hundred three thousandths

Converting a Decimal to a Fraction

To convert a decimal to a fraction you:

- 1) Determine the denominator by counting the number of digits to the right of the decimal point.
- 2) The numerator is the number to the right of the decimal point.
- 3) Reduce.

Example

- 1) Convert .52 to a fraction.

$$\begin{aligned} .52 &= \frac{52}{100} \\ &= \frac{13}{25} \end{aligned}$$

- 2) Convert .603 to a fraction.

$$.603 = \frac{603}{1000}$$

- 3) Convert 8.32 to a fraction.

$$\begin{aligned} 8.32 &= 8\frac{32}{100} \\ &= 8\frac{8}{25} \end{aligned}$$

Try a couple, convert these decimals to fractions.

1. .23
2. .511
3. .8
4. 13.51

Convert Fractions to Decimals

One way to convert fractions to decimals is by making equivalent fractions.

Example Convert $\frac{1}{2}$ to a decimal.

Since a decimal is a fraction whose denominator is a power of 10, I look for a power of 10 that 2 will divide into evenly.

$$\frac{1}{2} = \frac{5}{10}$$

Since the denominator is 10, I need only one digit to the right of the decimal point, the answer is .5

Example Convert $\frac{3}{4}$ to a decimal

Again, since a decimal is a fraction whose denominator is a power of 10, we look for powers of 10 that that will divide into evenly. 4 won't go into 10, but will go into 100.

$$\frac{3}{4} = \frac{75}{100}$$

There are denominators that will never divide into any power of 10 evenly. Since that happens, we look for an alternative way of converting fractions to decimals. Could you recognize numbers that are not factors of powers of ten? Using your Rules of Divisibility, factors of powers of ten can only have prime factors of 2 or 5. That would mean 12, whose prime factors are 2 and 3 would not be a factor of a power of ten. That means that 12 will never divide into a power of 10. The result of that is a fraction such as $\frac{5}{12}$ will not terminate – it will be a repeating decimal.

Because not all fractions can be written with a power of 10 as the denominator, we may want to look at another way to convert a fraction to a decimal. That is to divide the numerator by the denominator.

Example Convert $\frac{3}{8}$ to a decimal.

I could do this by equivalent fractions since the only prime factor of 8 is 2. However, we could also do it by division.

$$\begin{array}{r} .375 \\ 8 \overline{)3.000} \end{array}$$

Doing this problem out, we get 3.75

How do you know how many places to carry out the division? Your teacher would have to tell you.

1. $\frac{1}{4}$ 2. $\frac{3}{5}$ 3. $\frac{7}{8}$ 4. $\frac{5}{12}$

Remember, use common sense, if the fraction you are converting is less than one, then the decimal has to be less than one also.

Comparing Decimals

Since decimals are fractions, you compare decimals the same way you compare fractions. You find a common denominator and make equivalent fractions. The fraction with the largest numerator is the largest fraction.

Now, how does that work with decimals?

To compare decimal numerals you:

- 1) Write the decimals so that each decimal numeral has the same number of digits (add zeros).
- 2) Forget about the decimal points, the largest number will be the largest decimal numeral.

By adding zeros, you are finding a common denominator, just as you did when comparing fractions, now you can compare numerators. Isn't this neat the way this all comes together?

Examples: 1) Which is larger .8032 or .82

Add 2 zeros to .82 so both numbers will have 4 digits to the right of the decimal point.

.8032 and .8200 \longrightarrow both denominators are 10,000

since 8200 is larger than 8032, then $.82 > .8032$

2) Compare using $<, =, >$.62, .547

Add one zero to .62 so both numbers will have 3 digits. (denominator of 1000)

.620 and .547

620 is larger than 547, therefore $.62 > .547$

Use $>$ or $<$

1. .9, .235

2. 3.56, 9.1

3. .007, .7

Adding & Subtracting Decimals

Now, we get to add and subtract decimals. Remember, we defined decimals as fractions. We said all the rules for fractions should then work for decimals. Let's take a look at addition.

If I asked you to add $.72$ and $.126$, how would you do it?

Since decimals are fractions, I would use the fraction algorithm.

1. Find the common denominator
2. Make equivalent fractions.
3. Add the numerators
4. Bring down the denominator
5. Reduce

In that problem the denominator for $.72$ is 100, the denominator for $.126$ is 1000. The common denominator is 1000.

The way to make an equivalent fraction for $.72$ with a denominator of 1000 is by adding a zero to the end of the number. In other words, $.72 = .720$.

Now add the numerators, adding 720 to 126, I get 846. So far, so good.

Now using the fraction algorithm, how do I bring down the denominator of 1000. Well, in order to have a denominator of 1000, I have to have three digits to the right of the decimal point.

So in the number 846, where would I place the decimal point so we have a denominator of 1000? You've got it, before the 8, that would give us $.846$.

Adding $.72$ to $.126 = .846$.

That works because decimals are fractions. But, if we looked at enough addition or subtraction of decimals, we might see a pattern that would allow us to do the problem very quickly using a different algorithm.

Algorithm for Addition / Subtraction of Decimals

1. Rewrite the problems vertically, lining up the decimal points
2. Fill in spaces with zeros
3. Add or subtract the numbers.
4. Bring the decimal point straight down.

By lining up the decimal points and filling in zeros, I have done two things, I have found the common denominator and made equivalent fractions. When I added the numbers, I added the numerators. And by bringing the decimal point straight down, that accomplishes the same thing as bringing down the denominator. Just as we did with addition and subtraction of fractions. Isn't that neat!

Example $1.23 + .4 + 12.375$

Rewriting vertically

$$\begin{array}{r} 1.23 \\ .4 \\ +12.375 \\ \hline \end{array}$$

Now, filling in the zeros to find the common denominator and make equivalent fractions, then adding we have

$$\begin{array}{r} 1.230 \\ .400 \\ +12.375 \\ \hline 14.005 \end{array}$$

Notice, I brought the decimal point straight down. Bringing down the decimal point is analogous to bringing down the denominator.

Let's try a few of those.

1. $4.23 + .6 + 14.207$

2. $9.86 - 4.82$

3. $15.2 - 6.83$

4. $18.2 + 6 + .07$

5. $13.6 - 5.83$

6. $10 + 8.3 - 6.24$

Just as the algorithm for adding and subtracting decimals is related to addition and subtraction of fractions, the algorithm for multiplication of decimals also comes directly from the multiplication algorithm for fractions.

Before we see this analogy, let's first see the algorithm for multiplication of decimals.

Algorithm for Multiplication of Decimals

1. Rewrite the numbers vertically
2. Multiply normally, ignoring the decimal point
3. Count the number of digits to the right of the decimal points
4. Count that same number of places from right to left in the Product (answer)

Example 4.2×1.63

$$\begin{array}{r} 1.63 \\ \times 4.2 \\ \hline 326 \\ \underline{652} \\ 6846 \end{array}$$

Counting the number of digits to the right of the decimal points, I have two to the right in the multiplicand and one to the right in the multiplier – that's 3 altogether.

Now, we count that same number of places, 3, from right to left in our answer. That's where we put the decimal point.

So our answer is 6.846

Before going on, can you think how this procedure is related to the multiplication algorithm for fractions?

Remember, the algorithm for multiplying fractions, multiply the numerators, multiply the denominators, then reduce.

Well, when we multiplied the decimal, 4.2×1.63 , what we did by multiplying the numbers without regard to the decimal points was multiply the numerators.

Now looking at those two decimals, the denominators are 10 and 100 respectively. If we multiplied the denominators, we would end up with a denominator of 1000.

How many digits do we have to have to the right of the decimal point to have a denominator of 1000? Three zeros, three places.

Guess how many places we move the decimal point when we used the algorithm? You got it - 3.

The point being the algorithm for multiplication of decimals comes from the algorithm from multiplication of fractions. That should almost be expected since decimals are special fractions.

Again, we need to remember that if a number that does not have a decimal written, such as 15, the decimal point is understood to go after the number – 15.

Do you want some practice? Sure you do. Try some of these. Besides being able to do these problems, you should know the algorithm and you should understand that decimals are fractions.

1. $4.23 \times .6$
2. $526 \times .8$
3. $4.02 \times .106$
4. $.32 \times 09$
5. $7.23 \times .07$
6. 10.01×1.05
7. $73.4 \times .12$
8. 725.4×402

Multiplying by Powers of 10

Let's look at a couple of special cases for multiplication. By looking at a few of these problems, you will be able to multiply in your head. I know that excites you. I'm going to give you a number of multiplication problems just written with their answers. See if you see anything interesting develop?

$$10 \times 12.34 = 123.4$$

$$100 \times 567.234 = 56723.4$$

$$100 \times .0437 = 4.37$$

$$1000 \times 5.678 = 5678.$$

$$10 \times 3.579 = 35.79$$

$$100,000 \times 23.547892 = 2354789.2$$

Looking at the problem and looking at the answer, do you see anything? If you do, that pattern will lead us to another rule.

When you multiply by powers of 10, you move the decimal point to the right the same number of places as there are zeros.

Example 10×123.75

One zero in 10, move the decimal point one place to the right.

Therefore, $10 \times 123.75 = 1237.5$

Example 100×5.237

Two zeros in 100, move the decimal point two places to the right.

Therefore, $100 \times 5.237 = 523.7$

Example 1000×16.2

Three zeros in 1000, move the decimal point three places to the right. Therefore, $1000 \times 16.2 = 16200$. Notice I had to fill in a couple of placeholders to move it three places.

Now you try some.

1. 8.23×10 2. 8.23×100 3. 8.23×1000 4. 54.2×100

5. 100×154.3 6. 1000×5.6 7. 1.76×10^6 8. 8.543×10^2

Okay, we added, subtracted and multiplied, what do you think comes next?

Division of Decimals

If you answered, it's what amoebas have to do to multiply, you are so right. Yes, they have to divide. Don't you love a little math humor.

Algorithm for Dividing Decimals.

1. Move the decimal point as far to the right as possible in the divisor.
2. Move the decimal point the same number of places to the right in the dividend.
3. Bring up the decimal point straight up into the quotient.
4. Divide the way you normally would.

Example

$$.31 \overline{)25.834}$$

Move the decimal point 2 places to the right in the divisor.

$$31 \overline{)2583.4}$$

Move the decimal point 2 places to the right in the dividend.

Now, once you have moved the decimal points, you divide normally and bring the decimal point straight up.

$$\begin{array}{r}
 83.3 \\
 31 \overline{)2583.4} \\
 \underline{248} \\
 103 \\
 \underline{93} \\
 104 \\
 \underline{93} \\
 11
 \end{array}$$

By moving the decimal point the same number of places to the right in the divisor and dividend, what we are essentially doing is multiplying our original expression by one. In other words, we are making equivalent fractions by multiplying the numerator and denominator by the same number.

Let's look at that $\frac{25.834}{.31} \times \frac{100}{100} = \frac{2583.4}{31}$

If we move the decimal point one place, we are multiplying the numerator and denominator by 10. By moving it two places, we are multiplying the numerator and denominator by 100, etc.

1. $36 \overline{)194.4}$

2. $2.4 \overline{)36}$

3. $4.6 \overline{)1.288}$

4. $.26 \overline{)1302.6}$

5. $6 \overline{)3.00036}$

6. $.09 \overline{)1.872}$

Again, we should all know fractions and decimals are related. Decimals are special fractions whose denominators are powers of 10. Do you remember looking at special cases for multiplication of decimals? What we are going to do now is look at special cases for dividing decimals.

Dividing by Powers of 10

Again, what I am going to do is write some problems with their answers. See if you can see a pattern?

Examples

$$23.45 \div 10 = 2.345$$

$$346.853 \div 100 = 3.46853$$

$$87.23 \div 1000 = .08723$$

If we were to look at a few more problems, we might notice the digits in the answer stay the same as the digits in the problem, only the decimal points have moved. Hey, hey, hey, this identifying pattern stuff sure works out nice.

And as always, when we see a pattern that seems to work, we make up a shortcut or rule.

When you divide by powers of 10, you move the decimal point to the left the same number of places as there are zeros in the power of 10.

Example $345.8 \div 100$

Since there are two zeros, I move the decimal to the left 2 places. The answer is 3.458.

Example $87.239 \div 1000$

Three zeros, three places, the answer is .087239.

If you forget which way to move the decimal point, use common sense. When you divide by a power of 10, the quotient should get smaller, when you multiply, the product gets bigger.

1. $63.5 \div 100$

2. $1.874 \div 10$

3. $7.12 \div 1000$

4. $.832 \div 100$

5. 2.34×1000

6. $4.56 \div 100$

Scientific Notation

Very large and very small numbers are often written in scientific notation so numbers can be computed easily and as a means of saving space. Even calculators use scientific notation when computing with large or small numbers.

Scientific notation simplifies computing with very large or very small numbers, so its worth learning.

To write a number in scientific notation, you rewrite the number as a product of a number between one and ten and some power of ten.

The next example is contrived. Nobody would use scientific notation to write this number, but it is an easy enough example that will help us see how to rewrite numbers in scientific notation.

Example Write 420 in scientific notation.

I must rewrite 420 as a product of a number between one and ten and some power of 10.

$$_ \times 10^?$$

Where can I place the decimal point in 420 so it looks like a number between one and 10? Hopefully, you said between the 4 and 2. Let's see what we have.

$$4.20 \times 10^2$$

Since the decimal point is supposed to be to the right of zero, how many places will I have to move the decimal point to get it back to its original position? Two, therefore

$$420 = 4.20 \times 10^2$$

Example Write 96,000,000 in scientific notation

Place the decimal point between the 9 and 6, that gives us a number between one and ten

$$9.6 \times 10^?$$

To get the decimal point back to its original position, I would have to move the decimal point 7 places to the right, therefore the exponent is 7. Piece of cake!

$$96,000,000 = 9.6 \times 10^7$$

Write in scientific notation

1. 865,000,000,000

2. 170,000

Let's look at some small numbers.

Example Convert .00000234 to scientific notation

The decimal point goes between the 2 and 3 so we have a number between one and ten. After doing that, how many places do I move the decimal point to get it back to the original position? By counting, we see the decimal point has to be moved 6 places. . .

Since I am moving the decimal point to the left, the exponent is -6 . Therefore

$$.00000234 = 2.34 \times 10^{-6}$$

One way to remember if the exponent is positive or negative is that numbers greater than one have positive exponents, numbers less than one have negative exponents.

Write the following in scientific notation.

1. 744,000,000
2. 23,000,000
3. .00027
4. .00000000000000876

Write the following in standard form.

5. 6.23×10^7
6. 5.2×10^{12}
7. 3.24×10^{-8}
8. 7.4358×10^{-5}

Rewrite the following problem in scientific notation and simplify.

9.
$$\frac{72,000,000 \times 36,000}{180,000}$$

Rational & Irrational Numbers

A rational number is a number that can be written in the form $\frac{a}{b}$. Decimals are rational if they terminate or repeat because they can be written as a fraction. The following are examples of rational numbers.

$$8, \frac{3}{4}, .43, \bar{.3}$$

An irrational number cannot be written in the form $\frac{a}{b}$. Decimals are irrational if they are non terminating, and non repeating which can not be written as a fraction. The following are examples of irrational numbers.

$$\pi, e, \sqrt{5}, .313313331\dots$$

Simplifying Radicals (Square Roots)

If I asked you to find the $\sqrt{25}$, my guess is you would know the answer is 5. In fact, if I asked you to find the square root of numbers like 16, 25, 36, 49, or 100, you would probably know those answers as well because you know your multiplication facts. Why? Because they are perfect squares.

What would happen if I asked you to simply $\sqrt{50}$? My guess is you would run into some trouble. The fact is you don't know the $\sqrt{50}$. Is there any way to simplify that mathematically? You bet, otherwise I would not have brought it up.

To simplify a square root:

1. Rewrite the radicand as a product of a perfect square and some other number.
2. Take the square root of the perfect square.
3. Leave the other number in the radical.

Example Simply the $\sqrt{50}$

$$\begin{aligned}\sqrt{50} &= \sqrt{25 \times 2} \\ &= 5\sqrt{2}\end{aligned}$$

To simplify square roots quickly, you should be familiar with perfect squares. You find perfect squares by listing the Counting Numbers (1,2,3,...) and squaring them..

Perfect Squares

$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

$$6^2 = 36$$

$$7^2 = 49$$

$$8^2 = 64$$

$$9^2 = 81$$

$$10^2 = 100$$

Now, if I asked you to simplify the $\sqrt{300}$, you realize it was not a perfect square. So you would rewrite 300 as a product of a perfect square and some other number. Look at the table of perfect squares, which is the greatest factor of 300? Hopefully, you said 100. Therefore, we have

$$\begin{aligned}\sqrt{300} &= \sqrt{100 \times 3} \\ &= 10\sqrt{3}\end{aligned}$$

Simply the following square roots.

1. $\sqrt{20}$

2. $\sqrt{32}$

3. $\sqrt{72}$

4. $\sqrt{45}$

5. $\sqrt{98}$

Definitions

1. Decimal
2. Rational Number
3. Fraction
4. Write 16.237 in word form.
5. Write 32.405 in expanded notation
6. Write the algorithm for MULTIPLYING decimals
7. How are the algorithms for ADD/SUB fractions and decimals related?

8. Express in scientific notation

a. $645,000,000,000$

b. $.0000000468$

9. $12.345 + 8 + 2.03$

10. $13 - 5.67$

11. 52.4×6.1

12. $9.1 \overline{)20.762}$

13. Convert to a fraction

a) $.053$

b) $\overline{.23}$

14. Convert to a decimal

a) $3/8$

b) $1/9$

15 Round 54.374 to the nearest hundredth

16 Simplify

$$\frac{(4.2 \times 10^7)(8.1 \times 10^4)}{2.1 \times 10^3}$$

17. Simplify

a) $\sqrt{36}$

b) $\sqrt{300}$

18. Lenny went to the store and bought a chair for \$17.95, a rake for \$13.59, a spade for \$14.84, a lawn mower for \$189.99, and bags of fertilizer for \$3.29 each. What was his bill?

19. Bill's car travels 224 miles on 12 gallons of gas. How many miles to the gallon does his car get, rounded to the nearest mile?