Instruction matters!

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Mathematical Algorithms

A newsletter for middle school math teachers addressing best practices

An algorithm in mathematics is a step-by-step systematic procedure used in computation or problem solving.

The use of algorithms helps to address the difficulty students often have sequencing mathematical problems. The National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards* recommends that students use algorithms to compute and solve problems. However, algorithms should not stand-alone and usually need to be preceded by concept development.

Clearly, students should have an idea of how an algorithm was developed. Algorithms give students the general procedure that can be used to help them do their work more efficiently.

Most students have learned the following algorithm for adding or subtracting fractions.

Algorithm: Add/Sub Fractions

1. Find a common denominator
2. Make equivalent fractions
3. Add/subtract numerators
4. Bring down the denominator
5. Simplify

By having students memorize an algorithm, they are learning the vocabulary, learning how to communicate mathematically – acquiring the language, and the algorithm will help them perform the indicated operation.

If students can add/subtract fractions, they should also be able to put that into words, hence they will know the algorithm.

Knowing the algorithm will help students solve a problem. Knowing how to solve a problem will help students verbalize that knowledge in an algorithm.

Teachers showing students how to solve quadratic equations using the Zero Product Property would use an algorithm like this:

Algorithm: Solving Quadratic Equations

1. Write the polynomial on one side, zero on the other side of the equal sign
2. Factor completely
3. Set each factor equal to zero
4. Solve the resulting equations

Algorithms, procedures, are an important part of mathematics. The development of algorithms and the algorithm should be a regular part of mathematics instruction. Teachers should write algorithms on the board to help students compute or solve problems in math, the algorithms should be found in student notebooks, homework assignments, quizzes and tests. Students should be able to recite an algorithm to help them solve problems, explain the "why" behind mathematics and acquire the language of math.

Algorithms (shortcuts) were developed in many instances because someone recognized a pattern that would lead them to a desired result without having to do much work. Teachers should stress to students that the shortcuts, by themselves, often do not make sense. It is vital that students understand the concepts and how and why the shortcuts work.

With this knowledge, students are better able to make sense of math and are more likely to use suitable strategies to compute or solve problems Without algorithms, it will be clearly more difficult for students to compute or solve problems – to do math.

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A rational number (a/b) can always be represented by either a terminating decimal or as a repeating decimal. Using the Fundamental Theorem of Arithmetic, we know the only prime factors of a power of 10 are 2 and 5.

Knowing that, we can determine which fractions can be represented as terminating decimals and which fractions will not by inspection. Fractions whose denominators have only prime factors of 2 or 5 can be expressed as a terminating decimal. So fractions such as 2/5, 3/8, 7/10, 4/25, and 5/32 are all examples of fractions whose denominators only have prime factors of 2 or 5, thus could be written as a terminating decimal.

Fractions such as 7/12 will not terminate because the denominator has prime factors other than 2 or 5. The prime factors of 12 are 2 and 3. The only prime factors of powers of 10 (denominators for decimals) are 2 or 5. Since 3 is a factor of 12, we know that 12 will not divide into a power of 10 without a remainder. So fractions like 7/12 and 5/9 will not terminate, they will be repeating decimals.

How can you change a repeating decimal such a $.\overline{5}$ into a fraction?

We know how to convert .5 to a fraction, .125 to a fraction, what is different about converting $.\overline{5}$ to a fraction? Answer. It repeats.

As always in math, we look at the part of the problem that is causing us difficulty, then try to eliminate it. To eliminate the repeating part, we will use our knowledge of math (equations) and use the Properties of Real Numbers.

Algorithm –

Converting Repeating Decimals To Fractions

1. Let n = the repeating decimal
2. Multiply by a power of 10 so only the repeating part is to the right of the decimal point
3. Subtract the equations so the repeating parts are eliminated
4. Solve the resulting equation

Convert $.\overline{12}$ to a fraction.

1. Let n = $.\overline{12}$
2. Multiply both sides by 100 (2 numbers repeating)
   $100n = 12.\overline{12}$
3. Subtract the two equations
   $100n = 12.\overline{12}$
   $-\quad n = \quad .\overline{12}$
   $99n = 12$
4. Solve for n, $n = 12/99$

** To convert $.\overline{5}$ to a fraction, since only one digit is repeating, we'd multiply by 10^1. If three digits repeated, $.\overline{235}$, we'd multiply by 10^3. If you did enough of , you would see a pattern that would allow you to do these conversions in your head.

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