Solving Equations with Absolute Value

Bill Hanlon

Mathematics uses many symbols such as: +, –, x, and ÷. Another symbol we use is the ABSOLUTE VALUE sign.

Absolute Value is used when we want to concern ourselves with positive numbers. Finding a distance is a good example of using absolute values.

The absolute value of a number is defined to be a positive number. For example, the |5|, read the absolute value of 5, is POSITIVE 5. The |–6| = +6.

The number inside the absolute values signs could be positive or negative, but when you take the absolute value, the answer is positive.

Mathematically, that looks like this –

$$|x| = \begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x < 0 \end{cases}$$

When solving equations containing absolute values, we are looking for ALL values of the variable that will make the open sentence (equation) true. Just as we have done in the past.

EXAMPLE: Solve for x, $|x| = 8$

I am looking for values of x so the |x| is 8. The number inside the absolute values signs could be positive or negative. Clearly x could be 8 because the |8| is 8. But x could also be negative, x could equal –8 because the |–8| is 8.

Writing this out mathematically, we $x = 8$ or $-x = 8$, which is $x = -8$

We have two values of the variable that make the open sentence true, so x could be 8 or –8.

EXAMPLE: Solve for x, $|x - 1| = 10$

Let's reason this out a bit. The (x – 1) in the absolute value sign represents a number. That number, (x – 1), could be positive or it could be negative.

By definition, that means:

$x - 1 = 10$ or $-(x - 1) = 10$, which is equivalent to $x - 1 = -10$

If I solve those two equations, I will find the values of the variable that will make the open sentence true. One value will represent when (x – 1) is positive, the other when it is negative. Let's solve it.

$$\begin{array}{ccc} x - 1 = 10 & \text{or} & x - 1 = -10 \\ x = 11 & & x = -9 \end{array}$$

If we plug these values into the original equation, we see that they work.

If we did enough of these problems, we'd realize there would always be two answers. One solution will occur when the expression inside the absolute value sign is positive (greater than or equal to zero), the other solution will occur when that expression is negative.

EXAMPLE: Solve $|2x - 3| = 13$

$$2x - 3 = 13 \quad \text{or} \quad -(2x - 3) = 13 \text{ which is } 2x - 3 = -13$$

Solving each equation, we have

$$\begin{array}{rcl} 2x - 3 = 13 & \quad\text{or}\quad & 2x - 3 = -13 \\ 2x = 16 & & 2x = -10 \\ x = 8 & & x = -5 \end{array}$$

Let's plug those in to $|2x - 3|$ to check. If $x = 8$, then $2x - 3$ is $+13$ and the $|+13|$ is 13. That's true.

If $x = -5$, then $2x - 3$ is -13 and the $|-13|$ is 13. Both solutions work.

There are two solutions, $x = 8$ or $x = -5$.

Mathematically, we write the solution set in brackets $\{8, -5\}$.