Chapter 1

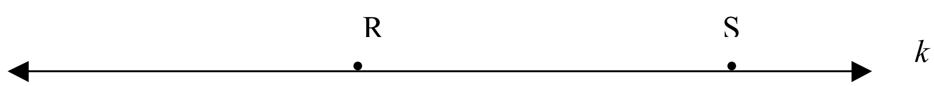
Points, Lines & Planes

As we begin any new topic, we have to familiarize ourselves with the language and notation to be successful. My guess that you might already be pretty familiar with many of the terms about to be introduced in this section, the biggest difference is that we will formalize our understanding and introduce notation that will enable to express that knowledge quickly. Let's look at one of our first elements in geometry, a point.

A *point* is pictured by a dot. While a dot must have some size, the point it represents has no size. Points are named by capital letters.

• **X**

A *line* extends indefinitely. A line, containing infinitely many points, is considered to be a set of points, hence it has no thickness. A line can be named by a lower case letter or by two points contained in the line.



This line could be called line k **or** $\overleftrightarrow{RS}$, read "line RS". Notice that $\overleftrightarrow{RS}$ does not begin or end at either of the points R or S.

A *plane* is a flat surface. Such things as table tops, desks, windowpanes, and walls suggest planes. A plane, like the aforementioned, does not have thickness and extends indefinitely.

The terms point, line, and plane are undefined terms. Other terms in geometry are defined. Notice the following definitions are based on the undefined terms.

Space- The set of all points.

Collinear points – A set of points that lie on one line.

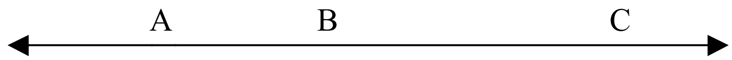
Coplanar points – A set of points that lie on a plane.

Axioms (postulates) – are basic statements, assumed without proof.

Theorems– are statements that are proven.

Subsets of a Line

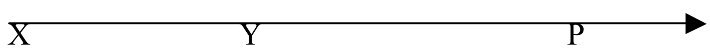
A point between two other points: Point B, on $\overleftrightarrow{AC}$ is said to be between points A and C and can be written as $\overleftrightarrow{ABC}$.



Segment: Given any two points A and B, segment AB is the set of points consisting of A and B and all the points that lie between A and B. Segment AB is denoted by $\overline{AB}$.

Segment Addition Postulate - A point between two other points: Point B, on $\overleftrightarrow{AC}$ is said to be between points A and C and can be written as $\overleftrightarrow{ABC}$ and AB + BC = AC

Ray: Ray XY, denoted by $\overrightarrow{XY}$, is the union of $\overline{XY}$ and the set of points P for which it is true that Y lies between A and P.



Opposite Rays: $\overrightarrow{SR}$ and $\overrightarrow{ST}$ are called opposite rays if S lies on $\overleftrightarrow{RT}$ between R and T.

Congruent Segments: Segments with equal lengths. If AB = XY, then AB is said to be congruent to XY, $\overline{AB} \cong \overline{XY}$

Midpoint of a segment: Point M is the midpoint of $\overline{RS}$ if M lies on $\overline{RS}$ and RM = MS.

Bisector of a segment: A line, segment, ray or plane that intersects $\overline{RS}$ at its midpoint bisects $\overline{RS}$ and is a bisector of $\overline{RS}$.

Symbols

$\overleftrightarrow{AB}$ - line containing A and B
$\overrightarrow{AB}$ - ray with endpoint A, passing through B
$\overline{AB}$ - segment joining A and B

AB - distance between A and B

It's very important that you take the time to memorize the notation for lines, line segments, rays, and distance. They will be used throughout the book and are meant to help you communicate mathematically. As we continue in our study, we will be introduced to more vocabulary and notation, if you learn it along the way, as it is introduced, then it will come naturally to you.

Three more terms that we will encounter quite often are axiom, theorem, and postulate.

Axiom (postulate) is a basic assumption in mathematics.

A **theorem** is a statement that is proved.

A **corollary** is a statement that can be proved easily by applying a theorem.

In a system of logic, math, the fewer assumptions (axioms) we make without proof, the better the system. But, we have to start somewhere, so let's look at some of our basic assumptions to begin our study of geometry.

Postulate A line contains at least two points, a plane contains at least three points not all on one line, and space contains at least four points not all in one plane.

Postulate Through any two points there is exactly one line.

Postulate Through any three points not on one line there is exactly one plane.

Postulate If two points lie in a plane, then the line joining them lies in the plane.

Postulate If two planes intersect, then their intersection is a line.

When a postulate or theorem is used quite frequently, we often give it a name. For instance, we introduced the **Segment Addition Postulate**, a postulate with a name because we will refer to again and again.

These postulates we accept as true because they seem to occur all the time and make sense to us. Based on these assumptions, postulates, we will then derive other information.

Theorem If two lines intersect, they intersect in exactly one point.

While that seems to make perfect sense, especially if we draw a picture, we call that a theorem rather than a postulate. The reason is, we can derive it, we can prove it.

Theorem If a point lies outside a line, then exactly one plane contains th point and the line.

Theorem If two lines intersect, exactly one plane contains both lines.

Ruler Postulate – The points on a line can be paired with the real numbers such that:

a. any point can be paired with zero
b. the distance between any two points is equal to the absolute value of the difference of the numbers paired with those points.

The Ruler Postulate suggests the next couple of theorems.

Theorem On a ray, there is exactly one point a given distance ***d*** from the endpoint of the ray.

Theorem A segment has exactly one midpoint.

We have introduced a lot of new terminology and notation so far in this chapter that you need to be comfortable using. We are not quite done.

Statements

We use statements like, "*If* you get your homework done before 6 pm, *then* we can watch television after dinner" all the time. In math, we refer to those statements as ***"if-then"*** statements.

If-then statements are called **conditional** statements. They are called conditionals because the end result depends on, is conditional on, that something was already done. A conditional in math is made up of two parts, the part that follows the if is called the **hypothesis**, it states what is given. The end result, the **conclusion**, is what we typically try to prove in geometry.

A conditional looks like this; If ***a***, then ***b;*** where ***a*** and ***b*** are both statements, ***a*** is the hypothesis (what is given to you), and ***b*** is the conclusion (what needs to be proved).

Mathematically, we would write that conditional as $A \to B$.

If you change the order of the hypothesis and conclusion, you can create another conditional. The new conditional is referred to as the **converse** of the original conditional.

That is; $A \to B$ is the statement, the converse of that statement is $B \to A$.

If it rains outside, then the sidewalks will be wet is an example of a conditional statement. If that turns out to be true, would the converse also be true? In other words, *if the sidewalks are wet, then it rained*, is that necessarily true? The answer is no, so be careful, don't think that just because a conditional statement is true, the converse has to be

Review Questions

1. In mathematics, basic assumptions are called

2. The distance between points D and E is represented by

3. If points A, B and C are noncollinear, a simple name for $\overline{AB} \cap \overline{BC}$ is

4. On a number line, the distance between two points with coordinates 3 and –5 is

5. If point T lies on $\overrightarrow{EB}$ but not on $\overline{EB}$, then the correct order of points B, E and T is

6. The statement "If AB = DF and DF = RS, then AB = RS" illustrates that the equality of numbers is

7. In the conditional "If r, then s," is called the

8. If a point A lies outside line ***t***, the number of planes that contain point A and line t is

9. On the number line, point D has coordinate 4 and point E has coordinate 12. Point X lies on ED and EX = 5. The coordinate of X is

10. The converse of the conditional "If q, then r" is

11. The conditional and the ______________________ are logically equivalent statements.