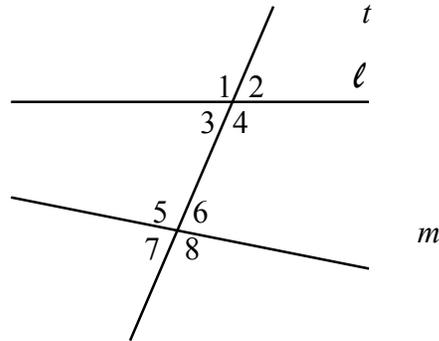


Chapter 3

Angles formed by 2 Lines being cut by a Transversal

Now we are going to name angles that are formed by two lines being intersected by another line called a transversal.



If I asked you to look at the figure above and find two angles that are on the same side of the transversal, one an interior angle (between the lines), the other an exterior angle that were not adjacent, could you do it?

$\angle 2$ and $\angle 4$ are on the same side of the transversal, one interior, the other is exterior – whoops, they are adjacent. How about $\angle 2$ and $\angle 6$?

Those two angles fit those conditions. We call those angles **corresponding** angles.

Can you name any other pairs of corresponding angles?

If you said $\angle 4$ and $\angle 8$, or $\angle 1$ and $\angle 5$, or $\angle 3$ and $\angle 7$, you'd be right.

Alternate Interior angles are on opposite sides of the transversal, both interior and not adjacent. $\angle 4$ and $\angle 5$ are a pair of alternate interior angles. Name another pair.

Based on the definition of alternate interior angles, how might you define **alternate exterior** angles?

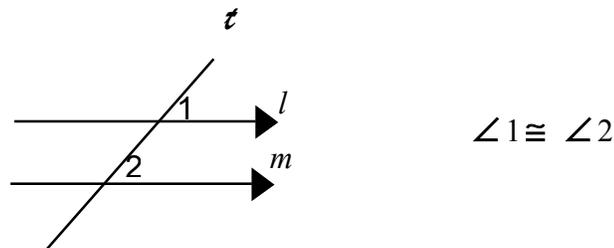
How about same side interior angles? Can you describe them? One pair of same side interior angles is $\angle 3$ and $\angle 5$, can you name another pair?

Angles Pairs formed by Parallel lines

Something interesting occurs if the two lines being cut by the transversal happen to be **parallel**. It turns out that every time I measure the corresponding angles, they turn out to be equal. You might use a protractor to measure the corresponding angles below. Since that seems to be true all the time and we can't prove it, we'll write it as an axiom – a statement we believe without proof.



Postulate If two **parallel** lines are cut by a transversal, the corresponding angles are congruent.



Now let's take this information and put it together and see what we can come up with.

Proofs: Alternate Interior Angles

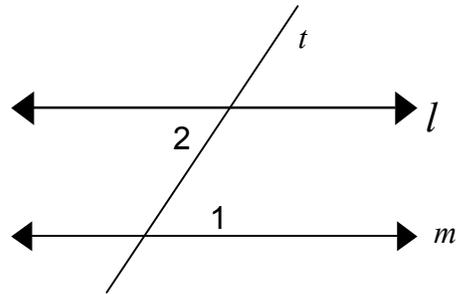
Let's see, we've already learned vertical angles are congruent and corresponding angles are congruent if they are formed by parallel lines. Using this information we can go on to prove alternate interior angles are also congruent if they are formed by parallel lines.

What we need to remember is drawing the picture will be extremely helpful to us in the body of the proof. Let's start.

Theorem

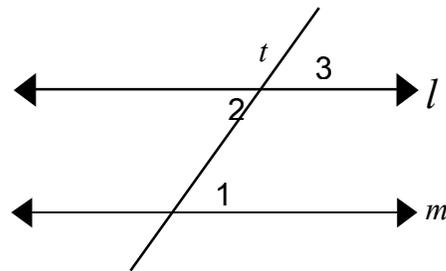
If 2 parallel lines are cut by a transversal, the alternate interior angles are congruent.

By drawing the picture of parallel lines being cut by a transversal, we'll label the alternate interior angles.



The question is, how do we go about proving $\angle 1 \cong \angle 2$?

Now this is important. We need to list on the picture things we know about parallel lines. Well, we just learned that corresponding angles are congruent when they are formed by parallel lines. Let's use that information and label an angle in our picture so we have a pair of corresponding angles.



Since the lines are parallel, $\angle 1$ and $\angle 3$ are congruent. Oh wow, $\angle 2$ and $\angle 3$ are vertical angles! We have studied a theorem that states all vertical angles are congruent.

That means $\angle 1 \cong \angle 3$ because they are corresponding angles and $\angle 2 \cong \angle 3$ are congruent because they are vertical angles, that means $\angle 1$ must be congruent to $\angle 3$.

$$\begin{aligned} \angle 1 &\cong \angle 3 \\ \angle 2 &\cong \angle 3 \end{aligned}$$

That would suggest that $\angle 1 \cong \angle 2$ by substitution.

Now we have to write that in two columns, the statements on the left side, the reasons to back up those statements on the right side.

Let's use the picture and what we labeled in the picture and start with what has been given to us, line l is parallel to m.

	Statements	Reasons
1.	$l \parallel m$ $\angle 1$ and $\angle 2$ are alt int \angle 's	Given
2.	$\angle 1$ and $\angle 3$ are corr. \angle s	Def of corr. \angle s
3.	$\angle 1 \cong \angle 3$	Two \parallel lines, cut by t, corr. \angle 's \cong
4.	$\angle 3 \cong \angle 2$	Vert \angle 's
5.	$\angle 1 \cong \angle 2$	Transitive Prop

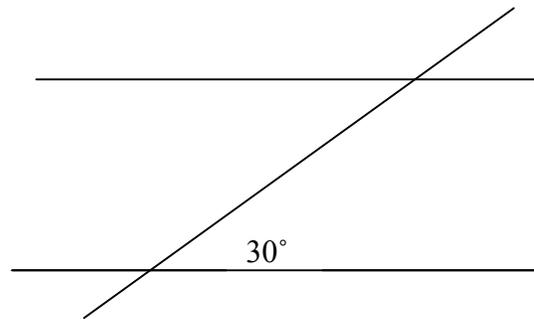
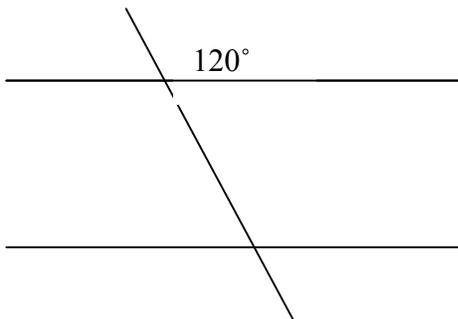
Is there a trick to this? Not at all. Draw your picture, label what's given to you, then fill in more information based on your knowledge. Start your proof with what is given, the last step will always be your conclusion.

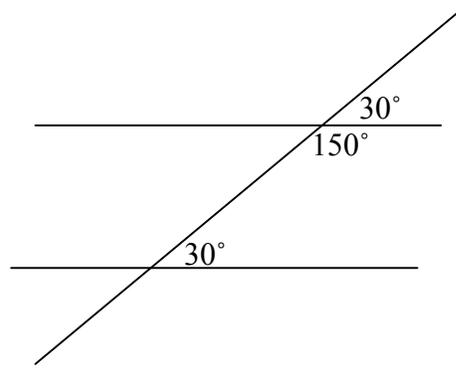
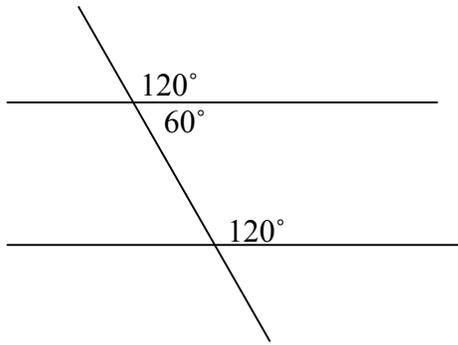
Now, we have proved vertical angles are congruent, we accepted corresponding angles formed by parallel lines are congruent, and we just proved alternate interior angles are congruent. Could you prove alternate exterior angles are congruent? Try it. Write the theorem, draw the picture, label the alternate exterior angles, add more information to your picture based on the geometry you know, identify what has been given to you and what you have to prove.

Let's write that as a theorem.

Theorem If two parallel lines are cut by a transversal, the alternate exterior angles are congruent.

If we played some more in the world of angles being formed by parallel lines, we might find an interesting relationship between the same side interior angles. Let's take a look.





If you filled in all the angles formed by those parallel lines being cut by a transversal, what relationship do you see when looking at the same side interior angles? Notice their sum is always 180°

Let's write that as a theorem.

Theorem If two parallel lines are cut by a transversal, the same side interior angles are supplementary.

Summarizing, we have:

If *two parallel lines* are cut by a transversal, congruent.

then the corresponding \angle 's are

then the alt. int. \angle 's are congruent.

then the alt. ext. \angle 's are congruent.

then the same side int. \angle 's are

congruent.

The good news is the converses of those statements are also true.

Showing lines are Parallel

Postulate If two lines are cut by a transversal so that the corresponding angles are congruent, then the lines are parallel.

This is the converse of the postulate that read; if two parallel lines are cut by a transversal, the corresponding angles are congruent. Now what I will accept as true is if the corresponding angles are congruent, the lines must be parallel.

The converse of a conditional is not always true, so this development is fortunate. As it turns out, the other three theorems we just studied about having parallel lines converses' are also true. That leads to the following three theorems.

Theorem If two lines are cut by a transversal so that the alternate interior angles are congruent, then the lines are parallel.

Theorem If two lines are cut by a transversal so that the alternate exterior angles are congruent, then the lines are parallel.

Theorem If two lines are cut by a transversal so that the same side interior angles are supplementary, then the lines are parallel.

Now I have four ways to show lines are parallel, corresponding \angle 's congruent, alternate interior \angle 's congruent, alternate exterior \angle 's congruent, same side interior \angle 's supplementary.

Indirect Proofs

There are times when trying to prove a theorem directly is either very difficult or impossible. When that occurs, we rely on our logic, our everyday experiences, to solve a problem. One such method is known as an *indirect proof*.

The logic of an indirect proof is based on the contra-positive of a conditional. You might remember a conditional such as $A \rightarrow B$. We said earlier that the converse of a that conditional is $B \rightarrow A$. We also mentioned that if the original conditional was true, that did not mean the converse was true.

A conditional and its contra-positive are logically equivalent statements. That is, if the conditional is true, then the contra-positive is also true. The contra-positive of the conditional $A \rightarrow B$ is $B' \rightarrow A'$.

Important to know – the contra-positive assumes the negation of the conclusion.

To begin an indirect proof, you suppose the negation of what you would like to prove is true. Then you reason logically until you encounter a contradiction of a known fact. You

then point out that the assumption in your opening statement is false and that it follows that the desired conclusion must be true.

Example Given: $\angle A$ is not congruent to $\angle B$

 Prove: $\angle A$ and $\angle B$ are not both right angles

Assuming the negation of the conclusion, we have $\angle A$ and $\angle B$ are right angles.

By theorem, we know all right angles are congruent, that is $\angle A \cong \angle B$ #

That contradicts the given fact that $\angle A$ is not congruent to $\angle B$. That means our assumption must be false and it follows that our desired conclusion is true. That is, since $\angle A$ and $\angle B$ are both right angles is false, then it follows that $\angle A$ and $\angle B$ are not both right angles.

Many people use the # to show where the contradiction appears in an indirect proof.

The success of an indirect proof depends upon finding a contradiction of a known fact. The known fact may be part of the hypothesis (given) of the statement to be proved, a postulate or theorem. Indirect proofs are also called “**Proofs by Contradiction.**”

Indirect proofs are especially useful when you want to prove there is exactly one of something. You assume there is more than one, then find a contradiction.