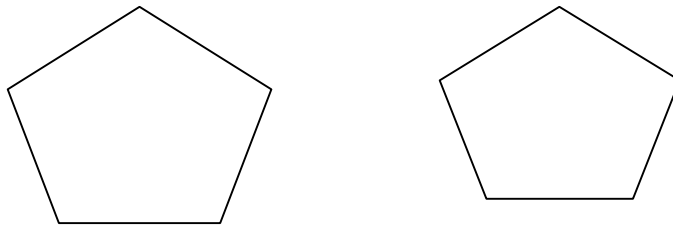


Chapter 7

Similar Polygons

When two polygons have the same shape and only differ in size, we say they are similar polygons.



These two pentagons are similar. More formally, two polygons are similar if and only if there is a one-to-one correspondence between their vertices such that:

1. the corresponding angles are congruent, and
2. the lengths of the corresponding sides are proportional

So just like congruence, similar polygons look the same, the corresponding angles are congruent, but their sides are not congruent, they are in proportion.

Since congruent polygons had the same shape and size, we used the congruence symbol - \cong . And we would write $\triangle ABC \cong \triangle DEF$.

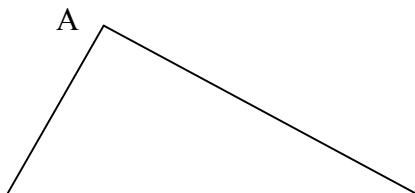
With similar polygons, the size changes so we no longer have the equality, so we take out the = signs in the congruence symbol and if triangles are similar, we would write $\triangle ABC \sim \triangle DEF$ and say triangle ABC is similar to triangle DEF.

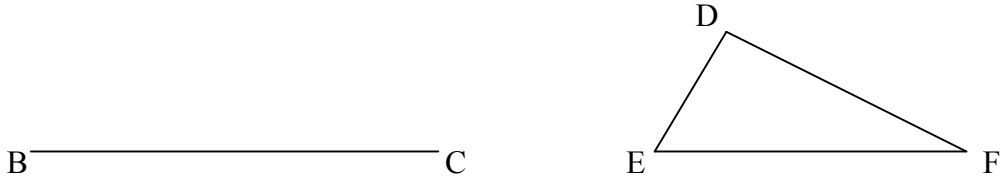
Taking this information with the properties of proportion we just learned, we learn more and be better able to apply the geometry we are learning.

Let's look at a theorem,

Theorem If two polygons are similar, the ratio of their perimeters equals the ratio of the lengths of any two corresponding sides.

As always, the best way to address any new information is by drawing and labeling a picture





$$\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle DEF} = \frac{AB}{DE}$$

To prove this theorem, all we have to do is use the last property of proportion we studied.

We know if the polygons are similar, the sides are in proportion. That is

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

The last property of proportion allows me to take the sum of the numerators and place that result over the sum of the denominators. Those sums are the perimeters of the two triangles.

Angle – Angle Postulate

Up to this point, the only way we could show triangles similar was by the definition. The next postulate is really going to make our lives easier with respect to triangles.

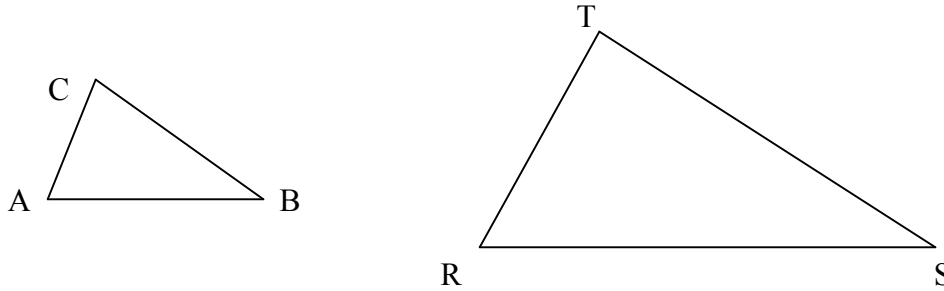
Postulate If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar.

The reason this is a postulate is that we cannot prove it, but as we examine more and more cases, this seems to hold up. Since it does, we accept it as true without proof.

Any postulate or theorem that has a name must be important, so make sure you know what this one says and means. Like \parallel theorems and postulates, this is a shortcut. Rather than showing all angles are congruent and the corresponding sides are in proportion, all I have to do is show two angles congruent.

Using this postulate, let's see what doors that can open to make more observations.

Theorem If an angle of one triangle is congruent to an angle of another triangle and the lengths of the sides including those angles are proportional, the triangles are similar.

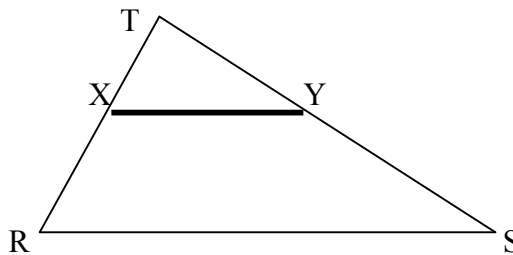


Given: $\triangle ABC$ and $\triangle RST$ with $\angle C \cong \angle T$ and $\frac{AC}{RT} = \frac{BC}{ST}$

Prove: $\triangle ABC \sim \triangle RST$

To prove that theorem, I will have to use a couple of strategies. The first is to draw and label the picture. The second is to bring in other information. I know the 2 angles are equal, that's a start. Now what I can do is pick a point X on TR so that $TX = CA$, then draw $XY \parallel RS$.

Let's see what that looks like and how that allows me to play with the geometry I already know.

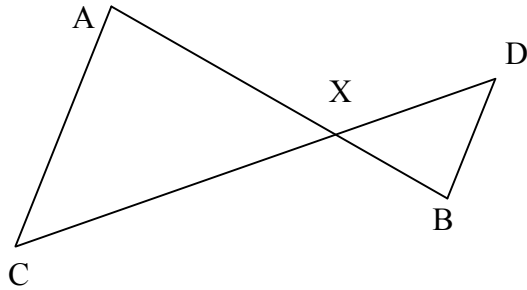


$\triangle TXY \sim \triangle TRS$ by the Angle-Angle Postulate, that results in

$$\frac{TX}{RT} = \frac{TY}{ST} \quad \text{Cross multiplying results in } TX \cdot ST = RT \cdot TY. \text{ By substituting } AC \text{ for } TX, \text{ we have } AC \cdot ST = RT \cdot TY$$

By the proportion given to us in the problem, we have $AC \cdot ST = RT \cdot BC$ by cross multiplying. Again, by substitution, we have $RT \cdot TY = RT \cdot BC$ or $BC = TY$ by dividing out the RT on each side. The rest follows easily.

Let's try another one.



Given: $\overline{AC} \parallel \overline{BD}$

Prove: $\triangle AXC \sim \triangle BXD$

	Statements	Reasons
1.	$AC \parallel BD$	Given
2.	$\angle A \cong \angle B$	2 \parallel lines cut by a transversal, alt int. angles are congruent
3.	$\angle AXC \cong \angle BXD$	Vertical angles are congruent
4.	$\triangle AXC \sim \triangle BXD$	Angle- angle Postulate

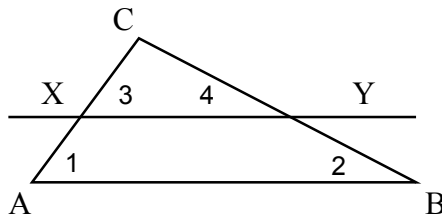
Once we know triangles are similar, we then know by definition the sides are in proportion. This will allow us to find lengths of sides of triangles.

Applications: Similar Δ 's

We learned the sides of similar polygons are in proportion. Using that information, we are able to solve problems.

If we continued in our study, we might be able to draw some more conclusions based on our observations.

Theorem If a line is parallel to one side of a triangle and intersects the other two sides, it divides them proportionally.



Given: ΔABC , $\overline{XY} \parallel \overline{AB}$

Prove: $\frac{AX}{XC} = \frac{BY}{YC}$

Statements	Reasons
1. $\overline{XY} \parallel \overline{AB}$	Given
2. $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$	\parallel lines cut by t. corr. angles
3. $\Delta ABC \sim \Delta XYC$	AA Postulate
4. $\frac{AC}{XC} = \frac{BC}{YC}$	Similar Δ 's
5. $AX + XC = AC$ $BY + YC = BC$	Segment Addition Thm
5. $\frac{AC - XC}{XC} = \frac{BC - YC}{YC}$	Prop of pro.
6. $AC - XC = AX$ $BC - YC = BC$	Sub. prop =.
7. $\frac{AX}{XC} = \frac{BY}{YC}$	Substitution

Following as a direct result of that theorem, we have the following corollary.

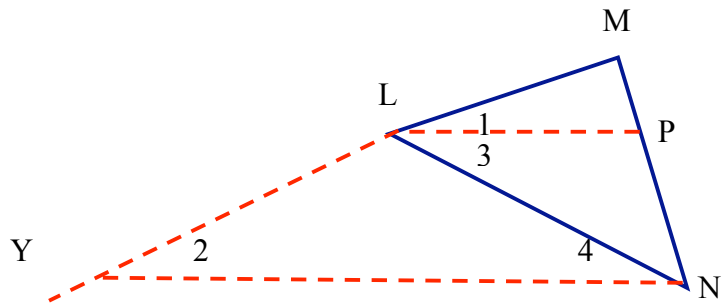
Corollary If three parallel lines intersect two transversals, they divide them proportionally.

Moving along, we have another theorem.

Theorem If a ray bisects an angle of a triangle, it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

Given: $\triangle LMN$
 LP bisects $\angle MLN$

Prove: $\frac{MP}{PN} = \frac{LM}{LN}$



Statements	Reasons
1. $\triangle LMN$ LP bisects $\angle MLN$	Given
2. Extend ML , $YN \parallel LP$	Construction
3. $\frac{MP}{PN} = \frac{LM}{LY}$ for $\triangle MYN$	line \parallel , divides \triangle pro
4. $\angle 1 = \angle 2$	Corr \angle 's
5. $\angle 1 = \angle 3$	Def \angle bisector
6. $\angle 3 = \angle 4$	Alt int \angle 's
7. $\angle 2 = \angle 4$	Sub
8. $LY = LN$	Base \angle 's \cong , opp. sides \cong
9. $\frac{MP}{PN} = \frac{LM}{LN}$	Sub in step 3

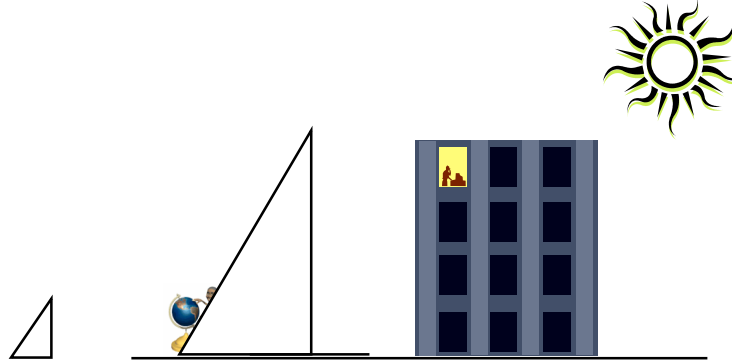
In this proof, we drew the picture, then we added on to that picture so we could use our previously learned knowledge of geometry.

Isn't this stuff neat? These theorems and postulates will allow us to find lengths of line segments using similar triangles.

Let's find the height of a building using similar triangles.

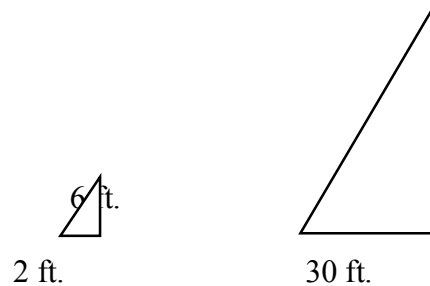
A 6 foot tall person casts a shadow 2 feet long. A building's shadow is 30 feet long, how tall is the building?

Drawing a picture, we have Mr. Man, Mr. Sun, and Mr. Building.



Because of the sun's distance from the earth, the top angles will be the same. We also have right angles, therefore by the AA Postulate, the triangles are similar.

$$\frac{\text{height of man}}{\text{length of shadow}} = \frac{\text{height of bldg.}}{\text{length of shadow}}$$



Just looking at the triangles without the man and building and filling in the numbers

$$2 \quad \frac{6}{3} = \frac{h}{30}$$

$$2h = 180$$

$$h = 90$$

The height of the building is 90 feet.