LOOK at Algebra 2

Basically you have learned how to solve and graph equations and inequalities.

You also learned many manipulative skills along the way. These skills allowed you to solve and graph those equations and inequalities very quickly.

The following pages are not meant for initial learning, but only to jog your memory about the concept & skills you learned in second year algebra.

In a nutshell, you have learned to solve:

1. Linear equations and inequalities
2. Systems of linear equations and inequalities
3. Quadratic equations and inequalities
4. Systems of quadratic equations and inequalities
5. Higher degree equations
6. Absolute value equations
7. Equations involving radicals
8. Exponential equations
9. Logarithmic equations

<u>Important Properties & Theorem</u>
Zero Product Property
Fundamental Theorem of Algebra
Remainder Theorem
Factor Theorem
Rational Root Theorem
DesCartes Rule of Signs

<u>Number Sets</u>
Natural #
Whole #
Integers
Rational #
Irrational #
Real #
Complex

How many of these formulas do you recognize, remember, and know how to use?

$c^2 = a^2 + b^2$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{midpt} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$y - y_1 = m(x - x_1)$

$y = mx + b$

$Ax + By = C$

$$m = \frac{-A}{B}$$

Parallel lines – same slope

Perpendicular lines – negative reciprocal slopes

$$x_{\text{int}} = \frac{C}{A} \qquad y_{\text{int}} = \frac{C}{B}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{\begin{aligned} ax^2 + bx + c &= 0 \\ r_1 + r_2 &= -b/a \\ r_1 r_2 &= c/a \end{aligned}}$$

$$y - k = \frac{1}{4m}(x - h)^2$$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$xy = k$

$$\log_b x = n \text{ iff } b^n = x$$

$$\log x_1 = \log x_2 \text{ iff } x_1 = x_2$$

$\log ab = \log a + \log b$ $\qquad$ $\log a/b = \log a - \log b$

$\log a^n = n \log a$ $\qquad$ $\log_b b^x = x$ $\qquad$ $b^{\log b^x} = x$

$y = kx$ $\qquad$ $xy = k$

$x^m x^n = x^{m+n}$ $\qquad$ $x^m/x^n = x^{m-n}$ $\qquad$ $(x^m)^n = x^{mn}$

$x^{-m} = 1/x^m$ $\qquad$ $x^0 = 1,\ x \neq 0$

$a_n = a_1 + (n-1)d$

$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$ $\qquad$ $a_n = a_1 r^{n-1}$

$= \frac{n}{2}\left(a_1 + a_n\right)$ $\qquad$ $S_n = \frac{a_1 - a_1 r^n}{1-r}$

$i = \sqrt{-1}$ $\qquad$ $i^2 = -1$ $\qquad$ $i^3 = -i$ $\qquad$ $i^4 = 1$

$\theta = \frac{s}{r}$ $\qquad$ $\frac{\deg rees}{radians} = \frac{180}{\pi}$

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

SOHCAHTOA

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\qquad$ $\cos^2 x + \sin^2 x = 1$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ $\qquad$ $c^2 = a^2 + b^2 - 2ab\cos C$

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos(A + B) = \cos A \cos B \mp \sin A \sin B$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$\tan(A + B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos\theta}{2}}$$

$$\tan \frac{\theta}{2} = \frac{\sin\theta}{1 + \cos\theta} = \frac{1 - \cos\theta}{\sin\theta}$$

$$P(n,r) = \frac{n!}{n - r)!}$$

$$C(n,r) = \frac{n!}{n - r)!r!}$$

$$(x + y)^n = \sum_{k=0}^{n} \frac{n!}{(n-k)!k!} x^{n-k} y^k$$

Linear Equations

Variable on one side, numbers on the other side – Order of Operations/reverse

a) $3x + 2x = 10$

$5x = 10$

$x = 2$

b) $4x - 2 = 10$

$4x = 12$

$x = 3$

c) $\frac{x}{2} - \frac{x}{3} = 1$

$6\left[\frac{x}{2} - \frac{x}{3}\right] = 6(1)$

$3x - 2x = 6$

$x = 6$

d) $\frac{x-2}{5} - \frac{x-3}{10} = 4$

$10\left[\frac{x-2}{5} - \frac{x-3}{10}\right] = 10(4)$ multiply by common dominator

$2(x - 2) - (x - 3) = 40$

$2x - 4 - x + 3 = 40$

$x - 1 = 40$

$x = 41$

Systems of Linear Equations

a) <u>Addition Method</u>

$3x + 2y = 6$

$\underline{6x - 2y = 12}$

$9x = 18$

$x = 2$

$\therefore \Rightarrow y = 0$; (2, 0)

b.) <u>Substitution</u>

$y = 2x + 1$

$\underline{2x + 3y = 19}$

$2x + 3(2x + 1) = 19$

$2x + 6x + 3 = 19$

$8x + 3 = 19$

$8x = 16$

$x = 2$

a1) <u>Mult-Add</u>

$3x + 2y = 5 \xRightarrow{x3} 9x + 6y = 15$

$4x + 3y = 6 \xRightarrow{x(-2} \underline{-8x - 6y = -12}$

$x = 3$

$\therefore \Rightarrow y = -2$ (3, –2)

$\therefore \Rightarrow y = 5$; (2, 5)

Quadratic Equations $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a) $12x^2 + 7x + 1 = 0$

a = 12
b = 7
c = 1

$$x = \frac{-7 \pm \sqrt{7^2 - 4(12)(1)}}{2(12)} = \frac{-7 \pm \sqrt{49 - 48}}{24}$$

$$= \frac{-7 \pm \sqrt{1}}{24}$$

$$= \frac{-7 \pm 1}{24}$$

$$x = \frac{-7 + 1}{24} \qquad x = \frac{-7 - 1}{24}$$

$$x = \frac{-6}{24} \qquad x = \frac{-8}{24}$$

---

b) $\dfrac{x-2}{x^2 - x - 6} = \dfrac{1}{x^2 - 4} + \dfrac{3}{2(x+2)}$

Factor denominators

(x–3)(x+2) (x+2)(x–2) ; The common dominator is 2(x+2)(x–2)(x–3)

Multiply both sides by the common dominator

2(x – 2)(x – 2) = 2(x – 3) + 3(x – 3)(x –2)

$$2(x^2 - 4x + 4) = 2x - 6 + 3(x^2 - 5x + 6)$$

$$2x^2 - 8x + 8 = 2x - 6 + 3x^2 - 15x + 18$$

$$0 = x^2 - 5x + 4$$

$$0 = (x - 4)(x - 1)$$

x = 4 x = 1

---

c) $x^2 - x - 12 \geq 0$ (inside – outside)

(x – 4) (x + 3) ≥ 0 ; x = 4 and x = –3 are the "zeroes" of the equality.

Plot those points on a number line, check any number between them. If that number makes the open sentence true, the answers is between those numbers. If it is false, it is outside those numbers.

0 is between –3 and 4 and does <u>not</u> work, therefore the answer is

x > 4 $\cup$ x < – 3 ; the numbers on the outside **or** Use Case I , Case II

Systems of Quadratic Equations

a) $y = x^2 + 2x + 3$

$y = 2x^2 + 5x + 7$ By Substitution

$$2x^2 + 5x + 7 = x^2 + 2x + 3$$
$$x^2 + 3x + 4 = 0$$
$$x = \frac{-3 \pm \sqrt{9 - 4(1)4}}{2(1)} = \frac{-3 \pm \sqrt{-7}}{2} = \frac{-3 \pm i\sqrt{7}}{2}$$

Now substitute and find y

b) $4x^2 + y^2 = 13$

$y = 4x^2 - 7$ By Addition

$$\begin{array}{l} 4x^2 + y^2 = 13 \\ \underline{-4x^2 + y = -7} \\ y^2 + y = 6 \end{array}$$

$$y^2 + y - 6 = 0$$
$$(y+3)(y-2) = 0$$

y = –3 y = 2 now substitute and find x

Higher Degree Equations - Rational Root Theorem

$x^3 + 4x^2 + 8x + 5 = 0$

by Rational Root Theorem

± 1 $\pm 1,\ \pm 5$; $\therefore$ the possible rational solutions are $\pm 1, \pm 5$

Using the factors theorem and synthetic division

$$\begin{array}{r|rrrr} -1 & 1 & 4 & 8 & 5 \\ & & -1 & -3 & -5 \\ \hline & 1 & 3 & 5 & 0 \end{array}$$

$\boxed{x = -1 \text{ is a solution}}$

1, 3, and 5 are the coefficients of the resulting quadratic equation

$$x^2 + 3x + 5 = 0$$
$$x = \frac{-3 \pm \sqrt{9 - 4(1)(5)}}{2}$$
$$= \frac{-3 \pm \sqrt{-11}}{2} = \frac{-3 \pm i\sqrt{11}}{2}$$

Absolute Value

$$|x| = \begin{cases} x, \ if \ x \geq 0 \\ -x, \ if \ x < 0 \end{cases}$$

Isolate the absolute value

a) $|6x - 2| = 40$

6x – 2 = 40	**or**	–(6x – 2) = 40
6x = 42		6x – 2 = –40
x = 7		6x = –38
		x = –38/6

b) $|5x - 3| > 27$

5x – 3 > 27	***or***	–(5x – 3) > 27
5x > 30		5x – 3 < – 27 **
x > 6		5x < – 24
		x < –24/5

c) $|2x + 3| \leq 11$

2x + 3 ≤ 11	***and***	–(2x + 3) ≤ 11
2x ≤ 8		2x + 3 ≥ – 11**
x ≤ 4		2x ≥ –14
		x ≥ – 7

$-7 \leq x \leq 4$

Radicals – isolate the radical

a) $2 - y - 2\sqrt{y+1} = 0$

$2 - y = 2\sqrt{y+1}$

$(2-y)^2 = \left(2\sqrt{y+1}\right)^2$

$4 - 4y + y^2 = 4(y+1)$

$4 - 4y + y^2 = 4y + 4$

$y^2 - 8y = 0$

$y(y-8) = 0$

$y = 0, \ y = 8$

<u>Check</u>, 8 does not work

the solution set is $\{0\}$

$\sqrt{x-5} = \sqrt{x} - 1$

Isolate & square

x - 5 = x- $2\sqrt{x}$+1

Isolate & square

$-6 = -2\sqrt{x}$

36 = 4x

9 = x

Exponential Equations

Rewrite in the <u>same</u> base

a)

$9^{x-1} = 81^{x+1}$

$\left(3^2\right)^{x-1} = \left(3^4\right)^{x+1}$

$3^{2x-2} = 3^{4x+4}$

$\therefore 2x - 2 = 4x + 4$

$-6 = 2x$

$-3 = x$

b)

$2^x + 2^x + 2^x + 2^x = 32$

$4 \bullet 2^x = 32$

$2^2 \bullet 2^x = 2^5$

$2^{2+x} = 2^5$

$\therefore\ 2 + x = 5$

$x = 3$

Logarithms

Use logs when you can't rewrite in same base

a)

$10^{2x-1} = 3.17$

$\log\ 10^{2x-1} = \log\ 3.17$

$2x - 1 = \log\ 3.17$

$2x - 1 = .5011$ (*see chart*)

$2x = 1.5011$

$x = 7505$

b)

$\log_{10} n + \log_{10} 5 = 1$

$\log_{10}\ 5n = 1$

$10^1 = 5n$

$2 = n$

c)

$10^{\log_{10} x^{2-1}} = 3$

$x^2 - 1 = 3$

$x^2 = 4$

$x = \pm 2$

d)

$3 \log x = \log 64$

$\log x^3 = \log 64$

$x^3 = 64$

$x = 4$

e)

$2^{x+y} = 32$; $2^{x+y} = 2^5$; $\Rightarrow$ $x + y = 5$

$2^{x-y} = 1$ $2^{x-y} = 2^0$; $\Rightarrow$ $x - y = 0$

$2x = 5$

$x = 5/2$

$y = 5/2$

Graphing Lines and Writing Equations of Lines $y - y_1 = m(x - x_1)$

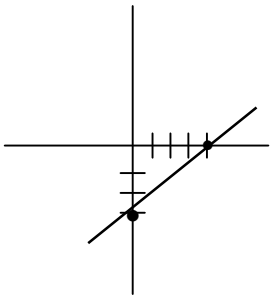
Graph

$3x - 4y = 12$

$m = \frac{3}{4}$

x int = 4

y int = -3

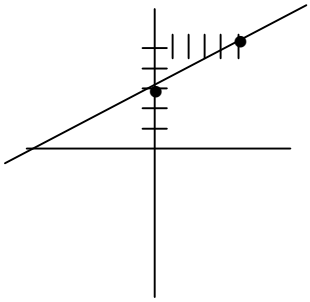


Graph

$y = \frac{2}{5}x + 3$

$m = \frac{2}{5}$

$y_{int} = 3$



Find an equation of a line passing through (2, 5) and (6, –1).

$y - y_1 = m(x - x_1)$ $\qquad$ $y - 5 = -\frac{3}{2}(x - 2)$

$m = \frac{\Delta y}{\Delta x} = \frac{5-(-1)}{2-6} = \frac{6}{-4} = -\frac{3}{2}$

Find an equation of a line passing through the point (–2, 5) parallel to 3x + y = 8.

$m = \frac{-3}{1} = -3$

$y - y_1 = m(x - x_1)$; $y - 5 = -3(x + 2)$

Find an equation of a line passing through the point (2, -7) perpendicular to $y = \frac{2}{5}x + 1$

m *of* $y = \frac{2}{5}x + 1$ is $\frac{2}{5}$

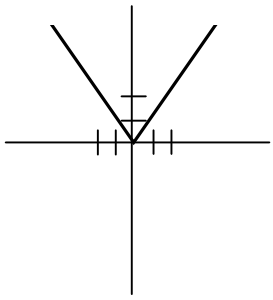
m of $\perp$ is $-\frac{5}{2}$

$y - y_1 = m(x - x_1)$; $y + 7 = -\frac{5}{2}(x - 2)$

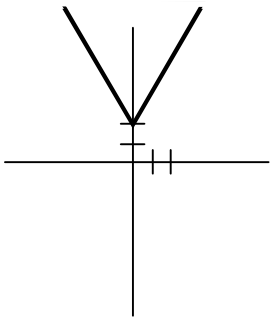
Graphs of Absolute Value

Graph

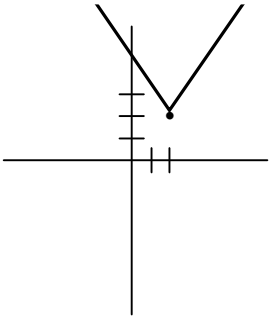
$y = |x|$



$y = |x| + 2$



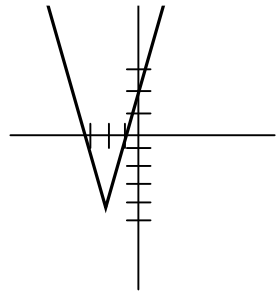
$y = |x\text{-}2| + 3$



Parabolas

y = ax^2 + bx + c

vertex$\left(-\frac{b}{2a},\ c-\frac{b^2}{4a}\right)$



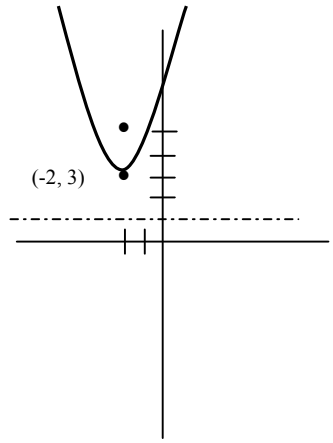
Graph

y = $2x^2$ + 8x + 3

v$\left(\frac{-8}{4},\ 3 - \frac{64}{8}\right)$

(-2, -5)

y – k = $\frac{1}{4m}(x-h)^2$



(h, k) - vertex

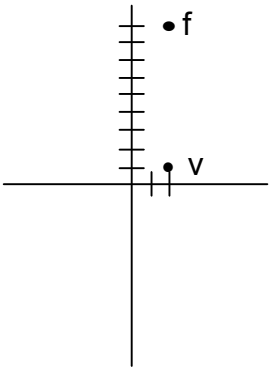
2m - distance from focus
To direrix

Graph $y-3=\frac{1}{8}(x+2)^2$

$\frac{1}{4m}=\frac{1}{8}$; m = 2

Therefore focus is at (-2, 5) and directrix is y = 1

Find an equation of a parabola with vertex at (2, 1) and focus at (2, 9)



$y-k=\frac{1}{4m}(x-h)^2$

$y-1=\frac{1}{4m}(x-2)^2$

distance from V to focus 8

$\therefore\ m = 8$

$y-1=\frac{1}{32}(x-2)^2$

Circles

$(x-h)^2+(y-k)^2-r^2$

Center (h, k)

radius r

Graph

$x^2+y^2-2x+6y=6$

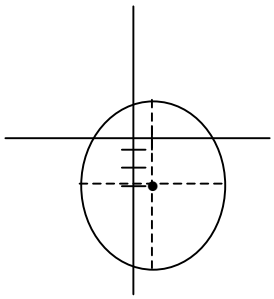
$x^2-2x+y^2+6y=6$

$x^2-2x+1+y^2+6y+9=6+1+9$

$(x-1)^2+(y+3)^2=4^2$

center (1, –3)

radius = 4



Find an equation of a circle with center (2, –5)
and passes through the point (1, 1)

$(x-h)^2+(y-k)^2=r^2$

$(x-2)^2+(y+5)^2=r^2$

$r=\sqrt{(2-1)^2+(-5-1)^2}$

$=\sqrt{1^2+6^2}=\sqrt{37}$

$(x-2)^2+(y+5)^2=\left(\sqrt{37}\right)^2$

$(x-2)^2+(y+5)^2=37$

Ellipses

The foci are always on the major axis

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

x int = $\pm a$ foci (c, 0), (–c, 0)

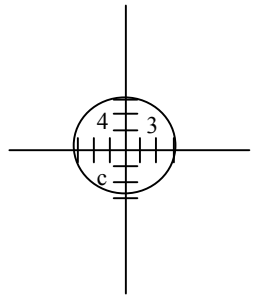
y int = $\pm b$

$$b^2+c^2=a^2$$

Sum of focal radius = 2a

Graph

$$\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$$



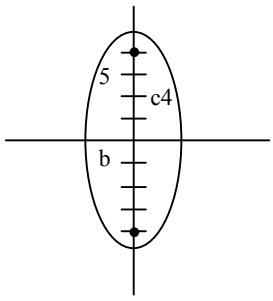
x int = ± 4

y int = ± 3

$$c^2+3^2=4^2$$
$$c^2=16-9$$
$$c=\sqrt{7}$$

foci ($\sqrt{7}$, 0), ($-\sqrt{7}$, 0)

Find an equation of an ellipse with the sum of the focal radius equal 10 and foci at (0, 4) and (0, -4).



Sfr = 2a

10 = 2a

5 = a

$$\frac{x^2}{}+\frac{y^2}{5^2}=1$$

$$c^2+b^2=a^2$$
$$4^2+b^2=5^2$$
$$b^2=25-16$$
$$b^2=9$$
$$b=3$$

$$\frac{x^2}{3^2}+\frac{y^2}{5^2}=1$$

Hyperbolas

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Difference of focal radius is 2a

vertices (a, 0), (–a,0)

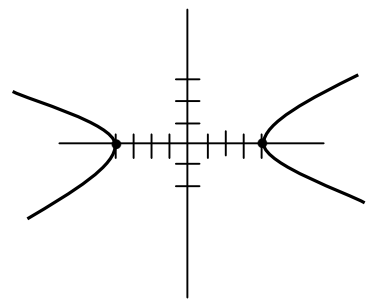
Asymptotic lines $y = \pm\frac{b}{a}x$

Foci (c, 0) and (–c, 0)

$c^2 = a^2 + b^2$

Graph

$\frac{x^2}{5^2} - \frac{y^2}{2^2} = 1$



$c^2 = 5^2 + 2^2$

$c^2 = 29$

$c = \sqrt{29}$

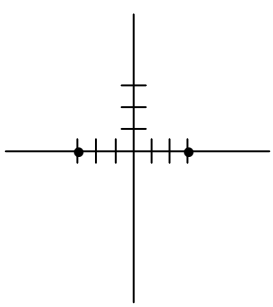
foci ($\sqrt{29}$, 0)

($-\sqrt{29}$, 0)

Asymptotic lines $y = \pm\frac{2}{5}x$

Find a equation of a hyperbola with vertices at

(3, 0) and (–3, 0) and asymptotic lines with $y = \pm\frac{2}{3}x$



$\frac{x^2}{3^2} - \frac{y^2}{\ } = 1$

$y = \pm\frac{b}{a}x$

$y = \pm\frac{2}{3}x$

$b = 2$

$\frac{x^2}{3^2} - \frac{y^2}{2^2} = 1$

Sequences & Series

Arithmetic Sequence (add)

The nth term $a_n = a_1 + (n - 1)d$

Find the 41st term of 5, 12, 19, . . .

$$a_{41} = a_1 + (41 - 1)\ 7$$

$$= 5 + (40)\ 7 = 285$$

Arithmetic Series

The sum of the 1st n terms

$$Sn = \frac{n(a_1 + a_n)}{2}$$

or by substitution

$$Sn = \frac{n}{2}(2a_1 + (n-1)d)$$

Find the sum of the 1st 41 term of 5, 12, 19, . . .

$$S_{41} = \frac{41}{2}\left[2(5) + (40)7\right]$$

$$= \frac{41}{2}(290)$$

$$= 41(145)$$

$$= 5945$$

Geometric Sequence (multiply)

The nth term $\mathbf{a_n = a_1 r^{n-1}}$

Find the nth term of 2, 4, 8, …

$$a_5 = 2 \text{ x } 2^{5-1}$$
$$= 2 \text{ x } 2^4$$
$$= 2 \text{ x} 16$$
$$= 32$$

Find the sum of the first 5 terms of 2, 4, 8, …

$$\mathbf{S_n = \frac{a_1(1-r^n)}{1-r} \; ; \; r \neq 1}$$

$$S_5 = \frac{2(1-2^5)}{1-2}$$

$$= \frac{2(1-32)}{-1}$$

$$= -2\,(-31)$$

$$= +62$$