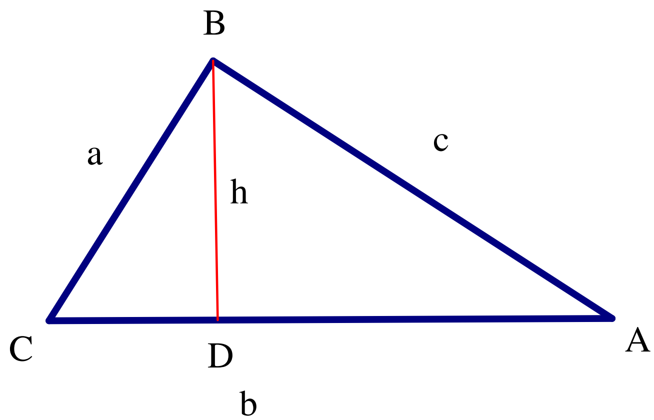
Derivation – Law of Cosines

Given ΔABC, construct an altitude (h), from $\angle B$ to $\overline{AC}$. That altitude forms two right triangles that allows us to use the trig.



$\sin C = \frac{h}{a}$ or $h = a \sin C$

Also, $\cos C = \frac{adj}{hyp}$

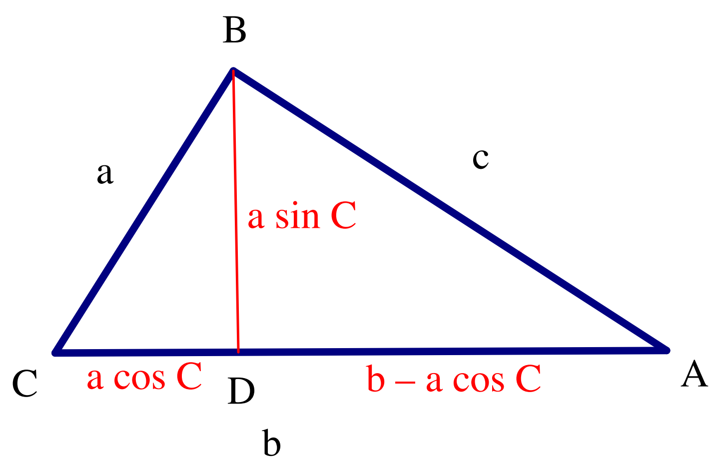
$\cos C = \frac{CD}{a}$

so $CD = a \cos C$

By the diagram AC = b and we just found that CD = a cos C, then

$$\mathbf{AD = b - a \cos C}$$

We will use that information to relabel our triangle ΔABC



ΔBDA is a right triangle, we can use the Pythagorean Theorem to write an equation

$$(a \sin C)^2 + \underline{(b - a \cos C)^2} = c^2$$

Squaring the binomial - underlined

$$a^2 \sin^2 C + \underline{b^2 - 2ab \cos C + a^2 \cos^2 C} = c^2$$

Rewriting the equation using the Commutative and associative properties that will result in a trig identity

$$a^2 \sin^2 C + a^2 \cos^2 C + b^2 - 2ab \cos C = c^2$$

Factoring a2 out of the first two terms

$$a^2 (\sin^2 C + \cos^2 C) + b^2 - 2ab \cos C = c^2$$

Substituting 1 for $\sin^2 C + \cos^2 C$

$$a^2 (1) + b^2 - 2ab \cos C = c^2$$

$$a^2 + b^2 - 2ab \cos C = c^2$$

OR

Law of Cosines

$$\mathbf{c^2 = a^2 + b^2 - 2ab \cos C}$$

In any triangle the square of a side equals the sum of the squares of the other two sides decreased by twice the product of those sides and the cosine of the included angle.

$$\mathbf{a^2 = b^2 + c^2 - 2bc \cos A}$$

$$\mathbf{b^2 = a^2 + c^2 - 2ac \cos B}$$