Derivation - Distance Formula

Using the Pythagorean Theorem, we can substitute coordinates to find the length of the hypotenuse. The length of the hypotenuse, the distance \( AB \) between the endpoints, will be identified by the letter \( d \) instead of \( c \).

Write vertices with ordered pairs

\[
\begin{align*}
A &= (x_1, y_2) \\
B &= (x_2, y_1) \\
C &= (x_1, y_1)
\end{align*}
\]

Use the ordered pairs to find the values of \( a \) (the horizontal distance) and \( b \) (the vertical distance).

\[
a = x_2 - x_1 \\
b = y_2 - y_1
\]

Let \( c \) be called \( d \)

Use the Pythagorean Theorem to find the hypotenuse and substitute the values of \( a \), \( b \), and \( c \).

\[
c^2 = a^2 + b^2
\]

Substitute for \( a \) and \( b \)

\[
d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2
\]
Take square root

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

**DISTANCE FORMULA**

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\(d\) is the distance between the points \(A(x_1, y_2)\) and \(B(x_2, y_1)\) or is the length of the segment \(AB\). \(AB\) can be interpreted as the length of the hypotenuse of the right triangle.

In short, the distance formula is the Pythagorean Theorem written differently to find the distance between two points or the length of a line segment.