

Derivation: Quadratic Formula Generalization of Completing the Square

1. $ax^2 + bx + c = 0$

Given

2. $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Div Prop =

3. $x^2 + \frac{b}{a}x = -\frac{c}{a}$

Sub Prop =

$\frac{1}{2} \left(\frac{b}{a}\right) = \frac{b}{2a}; \text{ then } \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

4. $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$

Add Prop =

5. $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

Factor/Com. Prop +

6. $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

CD

7. $\left(x + \frac{b}{2a}\right) = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$

Sq. Root, $x^2 = n$

8. $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

Sub Prop=/Simplify radical

9. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Add Fractions