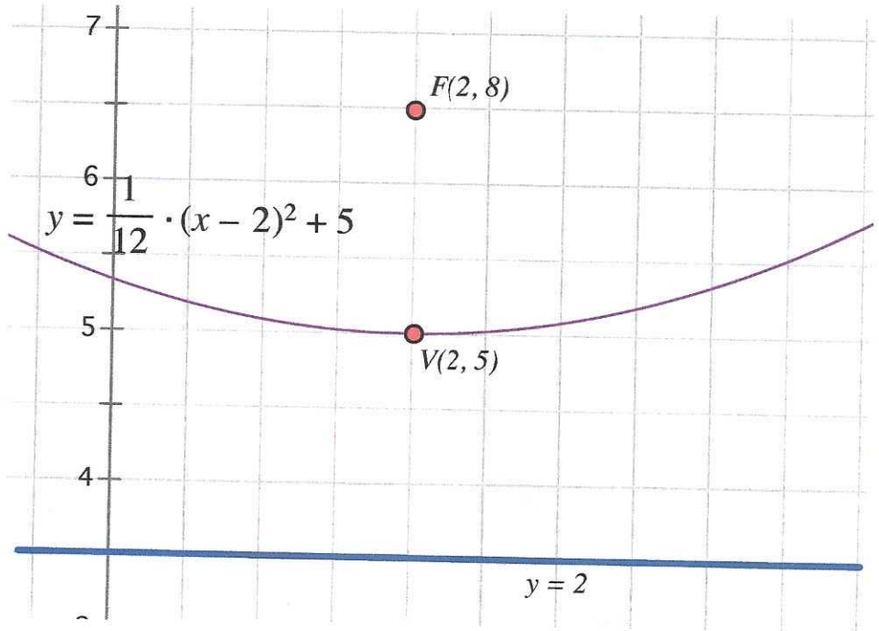
Example 1 Find the vertex, focus and the equation of the directrix for

$$y-5=\frac{1}{12}(x-2)^2$$

The vertex is at (2, 5), to find c, set $\frac{1}{4c}$ equal to the coefficient of the squared term - $\frac{1}{12}$. , therefore c = 3.

To find the focus, I add 3 to the y-coordinate of the vertex. So F(2, 8). The equation of the directrix is an equation of a line 3 down from the vertex; y = 2.



Example 2 Find an equation of a parabola with vertex (2, 3) and focus (2, 5).

To find the equation of a parabola, we need to know the vertex, which has been given to us, and the distance (c) from the vertex to either the focus or directrix.

$$y - 3 = (x - 2)^2$$

The distance from the V(2, 3) and F(2, 5) is 2, so $c = 2$.
Substituting $c = 2$ into the equation, we have

$$y - 3 = (1/8)[x - 2]$$

Example 3 Find an equation of a parabola with vertex (2, 1) that passes through the point (4, 13).

We need to find the values of ***a, h***, and ***k***. The Vertex Form of a parabola is $y = a(x - h)^2 + k$. Substituting the value of (h, k), we have

$y = a(x - 2)^2 + 1$.

Since we know the parabola passes through (4, 13), we can substitute those values into the equation and find the value of ***a***.

$13 = a(4 - 2)^2 + 1, \rightarrow 13 = 4a + 1$, so $a = 3$. Substituting the values of ***a, h*** an ***k*** back into vertex Form, we have

$y = 3(x - 2)^2 + 1$

Example 4 Find the vertex, focus, and directrix of $y - 2 = 4(x - 3)^2$

By inspection, we know this is a parabola whose vertex is at (3, 2) and since $4 > 0$, it opens up. To find c, I set $\frac{1}{4c} = 4$. Solving, we have $c = 1/16$.

The focus would be located at $(3, 2\frac{1}{16})$ and the equation of the directrix would be

$$y = 1\frac{15}{16}$$

I got those answers by adding 1/16 to the y-coordinate of the vertex and subtracting 1/16 from the y-coordinate of the vertex.

What would the equation in example 3 look like if I expanded the binomial?

$$y - 2 = 4(x - 3)^2 \qquad y - 2 = 4(x^2 - 6x + 9)$$

$$y - 2 = 4x^2 - 24x + 36$$

$$y = 4x^2 - 24x + 38$$

Note well! In this equation, only one of the variables has been squared. When this happens, we know the graph will be a parabola. To find the vertex, we will write it in vertex form.**Example 5** Find the vertex, focus, and directrix of $y = 3 - 6x - x^2$