Geometry, *You Can Do It !*
Pythagorean Theorem: Applications

by Bill Hanlon

To use the Pythagorean Theorem, we need to know how to simplify radicals. We'll look at simlifying square roots.

Simplifying expressions such as the $\sqrt{25}$ and the $\sqrt{64}$ are pretty straight forward.

$$\sqrt{25} = 5$$

$$\sqrt{64} = 8$$

The question that we need to consider is what happens if we want to take the square root of a number that is not a perfect square.

To simplify a square root, you rewrite the radicand as a product of a perfect square and some other number. You then take the square root of the perfect square.

If I square the numbers 1 through 10, the result will give me perfect squares.

Perfect Squares

1^2	=	1
2^2	=	4
3^2	=	9
4^2	=	16
5^2	=	25
6^2	=	36
7^2	=	49
8^2	=	64
9^2	=	81
10^2	=	100

Example

Simplify $\sqrt{50}$

Now 50 can be written as a product of 5 and 10. Should I use those factors ?

$$\sqrt{10 \cdot 5}$$

Hopefully, you said no. We want to rewrite the radicand as a product of a perfect square.

Neither 5 or 10 are perfect squares.
So, looking at my list of perfect squares, which, if any, are factors of 50 ?

That's right, 25 is a factor.

Simplifying, I now have

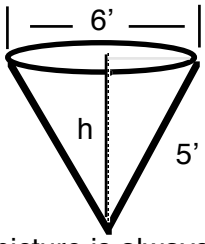
$$\sqrt{50} = \sqrt{25 \cdot 2}$$

$$= \sqrt{25} \cdot \sqrt{2}$$

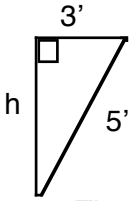
$$= 5\sqrt{2}$$

Sample Problem

Let's find the height of a cone knowing it's diameter is 6 feet and the slant height is 5 feet.



Drawing the picture is always helpful. I know the radius of the circle is half the diameter, that's 3'. I can also see a right triangle being formed.



Using the Pythagorean Theorem, I know the square of the hypotenuse is equal to the sum of the squares of the lengths of the other two legs.

Therefore, I have

$$5^2 = 3^2 + h^2$$
$$25 = 9 + h^2$$
$$16 = h^2$$
$$\sqrt{16} = h$$
$$4 = h$$

The height is 4 feet.

