Math, *you can do it!*

Exponentials, more

by Bill Hanlon

Last time we left off after developing two rules, one for multiplying exponentials, the other for dividing exponentials with the same bases.

Today, we'll build upon the definition of an exponent and the two rules we have already developed.

> Exponent - tells you how many times to use the base as a factor.
>
> Rule 1 - When you multiply exponentials with the SAME bases, you add the exponents.
>
> Rule 2 - When you divide exponentials with the SAME bases, you subtract the exponents.

Let's look at a problem like $9^2 \div 9^2$.

Since we are dividing exponentials with the same base, we subtract the exponents.

That leaves us with 9^0. Wow! That does not make sense. How can you use nine as a factor zero times?

That should lead us to believe that Rule 2 is not working. However, by looking at some more problems, we might be able to take care of that dilemma.

$5^2 \div 5^2 = 5^0$, $\quad 8^3 \div 8^3 = 8^0$, $\quad 4^5 \div 4^5 = 4^0$

Notice, in all these examples we are dividing a number by itself. A number divided by itself should be equal to one.

That would lead us to believe that $5^0 = 1$, $8^0 = 1$ and $4^0 = 1$.

That suggests a new rule.

> Rule 3. - Any number to the zero power, except zero, equals one.

The reason zero was excepted is because you can not divide by zero in the first place.

Look at a slightly different problem, $7^3 \div 7^5$.

Since we are dividing numbers with the same base, we use Rule 2 and subtract the exponents. The answer is 7^{-2}.

Whoops! That does not make sense! How can use 7 as a factor negative two times?

Going back to the definition, I have

$$\frac{7^3}{7^5} = \frac{\cancel{7 \times 7 \times 7}}{\cancel{7 \times 7 \times 7} \times 7 \times 7} = \frac{1}{7^2}$$

Try a couple more examples on your own. Notice a pattern?

Example Simplify $5^4 \div 5^7 = 5^{-3}$

By definition, we have

$$\frac{5^4}{5^7} \qquad \frac{\cancel{5 \times 5 \times 5 \times 5}}{\cancel{5 \times 5 \times 5 \times 5} \times 5 \times 5 \times 5} = \frac{1}{5^3}$$

Hopefully, you are noticing that $7^{-2} = \frac{1}{7^2}$ and $5^{-3} = \frac{1}{5^3}$ That might lead us to Rule 4

> Rule 4 - A number raised to a negative exponent means one over that number to a positive exponent.

Example Simplify $8^3 \div 8^4$

$$= 8^{3-4}$$
$$= 8^{-1}$$
$$= \frac{1}{8}$$

See if you can't figure out why this next rule works.

> Rule 5 - When you raise a power to a power, you multiply the exponents.

Example Simplify $(5^4)^2$

$$(5^4)^2 = 5^4 \times 5^4$$
$$= 5^8$$

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