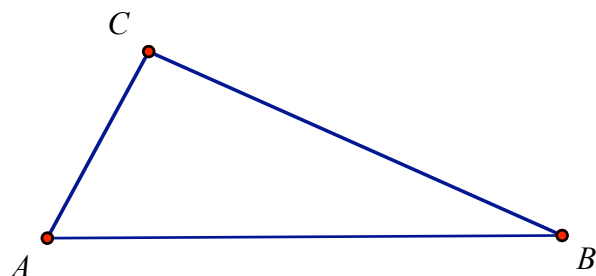


Chapter 6

Triangle

A triangle (Δ) is the union of three segments determined by three noncollinear points.



Each of the three points, A, B and C is a **vertex** of the triangle.

\overline{AB} , \overline{BC} , and \overline{AC} are called the **sides** of the triangle. $\angle A$, $\angle B$, and $\angle C$ are called the **angles** of the triangle.

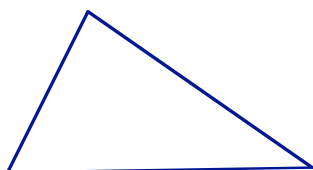
As there are parts to a triangle, we often look at different classifications of triangles for convenience in describing them. Sometimes they are classified by the number of congruent sides, sometimes by the number of congruent angles.

Triangle Classification by sides

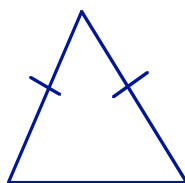
A **scalene** Δ is a Δ with no two sides \cong .

An **isosceles** Δ is a Δ with at least two sides \cong .

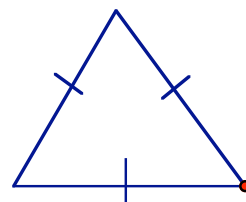
An **equilateral** Δ is a Δ with all sides \cong .



scalene Δ



isosceles Δ



equilateral Δ

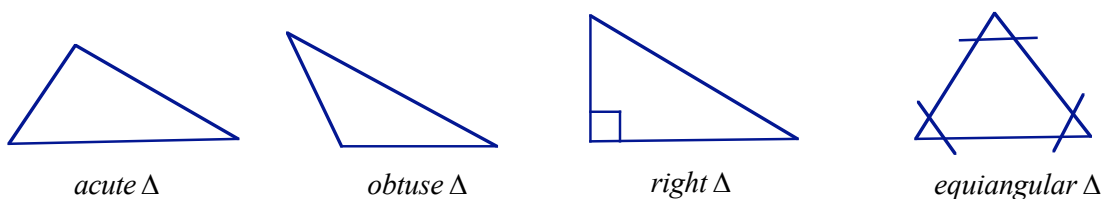
Triangle Classification by angles

An **acute** Δ is a Δ with three acute \angle s.

An **obtuse** Δ is a Δ with an obtuse \angle s.

A **right** Δ is a Δ with a right \angle .

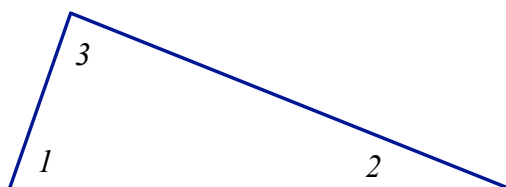
An **equilateral** Δ is a Δ with all \angle s \cong .



Angle Theorems: Triangles

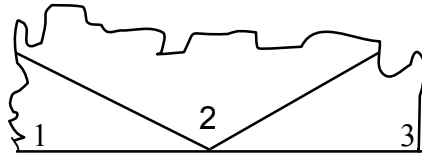
If I asked an entire class to draw a triangle on a piece of paper, then had each person cut out their triangle, we might see something interesting happen if they tore out the angles and placed them side by side.

Let's label the angles 1, 2, and 3 as shown.



First, we will cut the triangle out. Hopefully everyone's triangle will look different. Some might be scalene, right, or equilateral triangles. After cutting each triangle, we will tear each angle from the triangle.

Placing those angles side by side, the three angles always seem to form a straight line. Neato!

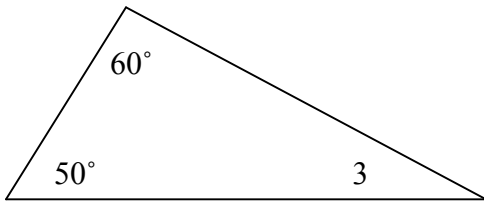


Since that would be occurring for everyone that drew the triangle, that might lead me to believe the sum of the three interior angles of a triangle is 180° .

While that's not a proof, it does provide me with some valuable insights. The fact is, it turns out to be true, so we write it as a theorem.

Theorem The sum of the interior angles of a triangle is 180° .

Example: Find the measure of $\angle 3$.



Since the sum of the two angles given is 110° , $\angle 3$ must be 70°

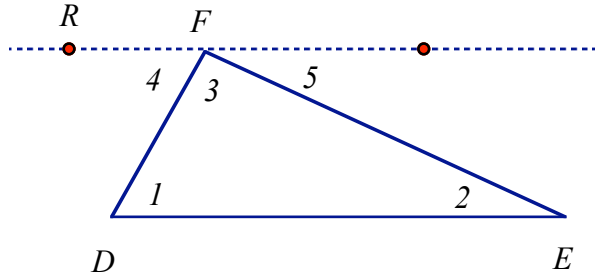
Drawing a triangle and cutting out the angles only suggests the sum of the interior angles of a Δ is 180° . It is not a proof. Let's see what that proof might look like.

Now the importance of knowing your theorems, postulates and definitions. If all I were to do was draw a triangle and label the three angles, I would not get far with the proof. What I need to do is use all the information I have learned before and apply it to this theorem.

Before this chapter we were working with parallel lines. While triangles are certainly not made up of parallel lines, I can use my knowledge of geometry and draw a line parallel to the base of a triangle. I will use the reason as construction. But, we know, from a point not on a line, there is exactly one line parallel to that line through that point.

By introducing parallel lines, I will be able to use the relationships we learned previously about angles being congruent. You just gotta love math! Let's look at the proof of this theorem.

Theorem The sum of the measures of the interior angles of a triangle is 180° .



Given: $\triangle DEF$

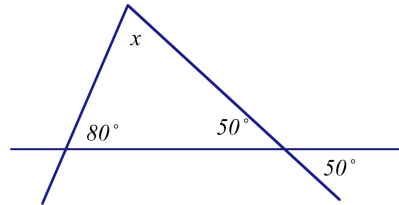
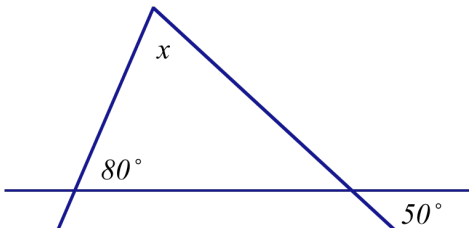
Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$

The most important part of this proof will be our ability to use the geometry we have already learned. If we just looked at the three angles of the triangle, we'd be looking for an awfully long time without much to show for it. What we will do is use what we just learned – we were just working with parallel lines, so what we will do is put parallel lines into our picture by constructing \overline{RS} parallel to \overline{DE} and labeling $\angle 4$ and $\angle 5$ formed. Now we have parallel lines being cut by transversals, we can use our knowledge of angle pairs being formed by parallel lines.

Statements	Reasons
1. $\triangle DEF$	Given
2. Draw $RS \parallel DE$	Construction
3. $\angle 4 \wedge \angle DFS$ are supp	Ext sides of 2 adj \angle 's - line
4. $\angle 4 + \angle DFS = 180^\circ$	Def Supp \angle 's
5. $\angle DFS = \angle 2 + \angle 5$	\angle Add Postulate
6. $\angle 4 + \angle 2 + \angle 5 = 180^\circ$	Substitution
7. $\angle 1 = \angle 4$ $\angle 3 = \angle 5$	2 \parallel lines cut by t, alt int \angle 's =
8. $\angle 1 + \angle 2 + \angle 3 = 180^\circ$	Substitution

Notice how important it is to integrate our knowledge of geometry into these problems. Step 3 would not have jumped out at you. It looks like angles 4, 2, and 5 form a straight angle, but we don't have a theorem to support that so we have to look at $\angle 4$ and $\angle DFS$ first.

Example: Find the value of x .

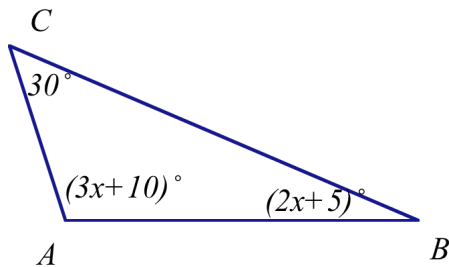


Notice in the diagram on the right, we were able to fill in one of the angles of the triangle because we know vertical angle are congruent.

Now, using the last theorem, the sum of the interior angles of a triangle measure 180° , we have

$$\begin{aligned} 80^\circ + 50^\circ + x &= 180^\circ \\ 130^\circ + x &= 180^\circ \\ x &= 50^\circ \end{aligned}$$

Example: Find the $m\angle A$



We know $m\angle A + m\angle B + m\angle C = 180^\circ$.

Use that to find the value of x , then substitute that in the expression representing $m\angle A$

$$\begin{aligned} (3x+10)^\circ + (2x+5)^\circ + 30^\circ &= 180^\circ \\ 5x + 45 &= 180 \\ 5x &= 135 \\ x &= 27^\circ \end{aligned}$$

$$m\angle A = (3x+10)^\circ \rightarrow m\angle A = 3(27) + 10 = 91^\circ$$

If we did a few more problems with this theorem, *the sum of the interior angles of a triangle measures 180°* , we would see some other relationships very quickly. When we see relationships that seem to follow directly from a theorem, we call those **corollaries**.

Corollary 1 If 2 \angle s of one Δ are \cong to 2 \angle s of another Δ , the third \angle s are \cong .

We will use this corollary quite frequently. This follows directly from the sum of the three interior angles must be 180° .

Corollary 2 The measure of each \angle of an equilateral Δ must be 60° .

Again, if the sum is 180° , then each equal \angle must have measure 60° .

Corollary 3 In a Δ there can be at most one right \angle or one obtuse \angle .

If there were more than one right or obtuse angle, the sum would be greater than 180° .

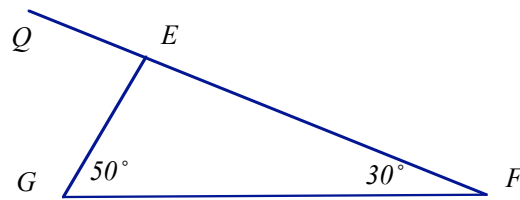
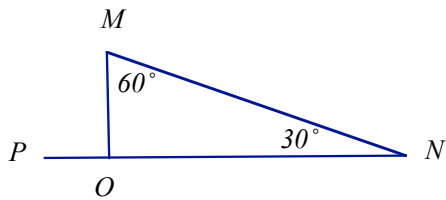
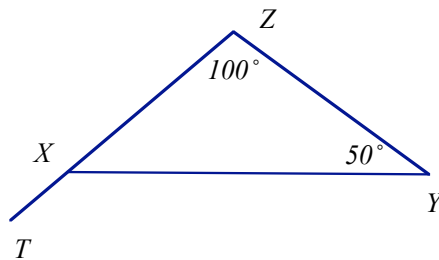
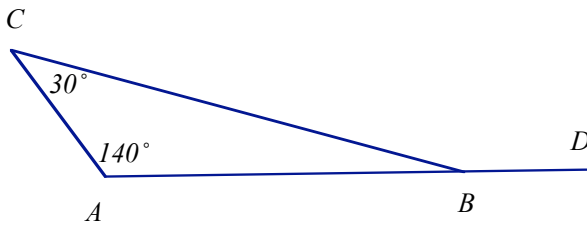
Corollary 4. The acute angles of a right Δ are complementary.

Still again, this follows because the sum of the three \angle s must be 180° , if one of the \angle s is 90° , then the sum of the other two \angle s must be 90° .

A theorem that seems to follow from that theorem is one about the relationship between the exterior angle of a triangle and angles inside the triangle. If we drew three or four triangles and labeled their interior angles, we would see a relationship between the two remote interior angles and the exterior angle.

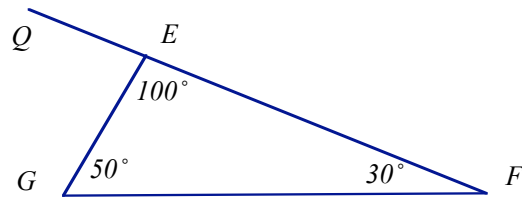
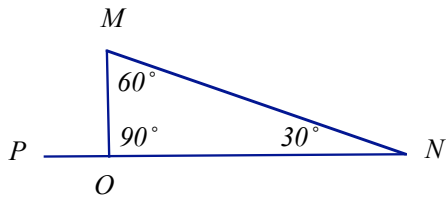
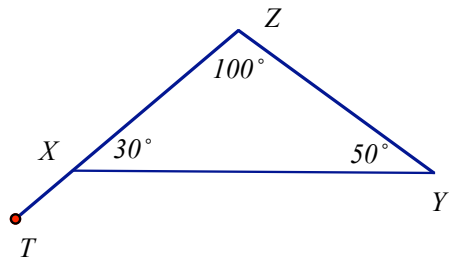
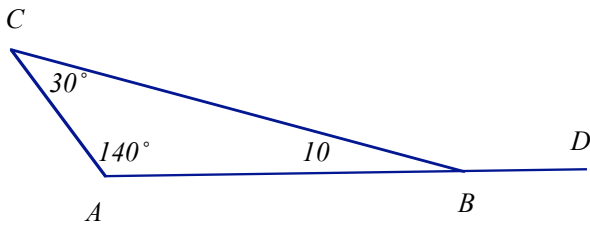
Let's take a look at a few triangles and see if we can see the relationship.

Examples: Find the missing measure of each of the following triangles.



In ΔABC , $m\angle B = 10^\circ$. ΔXYZ , $m\angle X = 30^\circ$, ΔMON , $m\angle O = 90^\circ$, and in ΔEFG , $m\angle E = 100^\circ$

We can find those measures because we know the sum of the interior angles of a triangle measure 180° . If we can find those angles, can we find the measure of the exterior angles drawn. See any relationship between those angles?



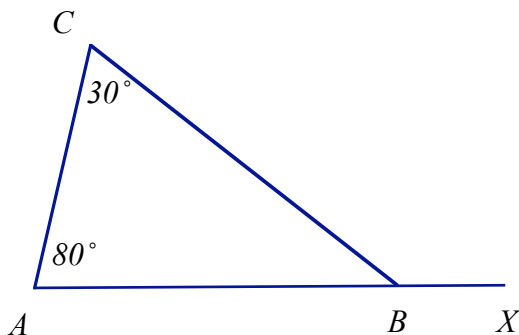
Filling in the measures of each of the angles we have found, we can find the measure of each exterior angle drawn by knowing a previous theorem, *if the exterior sides of two adjacent angles lie in a line, the angles are supplementary – measure 180° .*

That would mean $m\angle CBD = 170^\circ$, $m\angle TXY = 150^\circ$, $m\angle MOP = 90^\circ$, and $m\angle GEQ = 80^\circ$.

So, not only could we find the missing third angles, we could find the measure of the exterior angles given to us in the diagrams.

If we examined these problems long enough, we might also see a relationship between the exterior angle and the interior angles of a triangle. The relationship is the measure of the exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles.

Example: Find the measure of the exterior angle, $m\angle CBX$.



Using our observation, $m\angle CBX = 110^\circ$, the sum of the two non-adjacent angles.

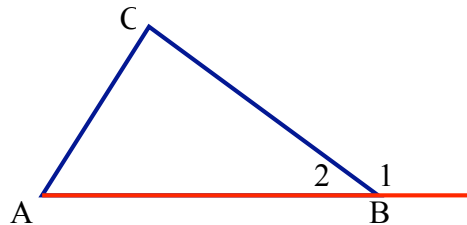
Or you could have done it the long way, find the $m\angle B$, then use supplementary angles to find $m\angle CBX$. So, what we will do is use two relationships, the sum of the interior angles of a triangle is 180° and if the exterior sides of two adjacent angles lie in a line, they are supplementary. Since both

relationships equal 180° , we will set them equal and solve the resulting equation. Now, let's look at the proof.

Theorem: The exterior \angle of a Δ is equal to the sum of the 2 remote interior \angle 's

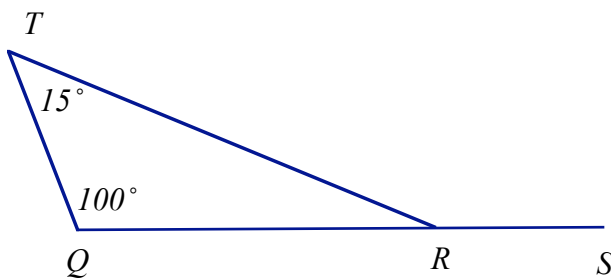
Given: ΔABC

Prove: $m\angle 1 = m\angle A + m\angle C$



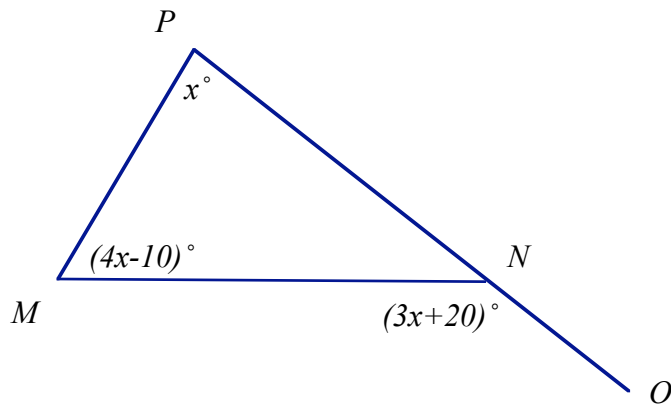
Statements	Reasons
1. $m\angle A + m\angle C + m\angle 2 = 180^\circ$	Int \angle 's of $\Delta = 180^\circ$
2. $\angle 1 \wedge \angle 2$ are supp \angle 's	Ext sides 2 adj \angle 's - line
3. $m\angle 1 + m\angle 2 = 180^\circ$	Def Supp \angle 's
4. $\angle A + \angle C + \angle 2 = \angle 1 + \angle 2$	Substitution
5. $m\angle A + m\angle C = m\angle 1$	Sub Prop Equality

Example: Find the $m\angle TRS$



By the last theorem,
 $m\angle TRS = m\angle T + m\angle Q$
 $m\angle TRS = 15^\circ + 100^\circ$
 $m\angle TRS = 115^\circ$

Example: Find the $m\angle M$



To find $m\angle M$, we have to find the value of x , then substitute that back into the expression representing $m\angle M$.

Using the theorem, the ext \angle of a Δ is = to the sum of the 2 remote int \angle s, we have

$$\begin{aligned}m\angle MNO &= m\angle M + m\angle P \\3x + 20 &= 4x - 10 + x \\3x + 20 &= 5x - 10 \\30 &= 2x \\15 &= x\end{aligned}$$

If $x = 15$ and $m\angle M = (4x-10)^\circ$, then $m\angle M = 4(15) - 10 = 50^\circ$